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Sovereign Default, Taxation, and the Underground Economy

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Sovereign Default, Taxation, and the Underground Economy[*]

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Abstract

This paper studies the dynamic interaction between sovereign default risk, taxation, and the underground economy. For a large sample of countries, we find that the size of the underground economy is positively correlated with sovereign debt and interest spreads. We rationalize these empirical regularities within a quantitative model of sovereign default that explicitly accounts for underground activities. We highlight a vicious circle: Higher sovereign risk premia tighten the endogenous borrowing constraint and force the government to raise taxes. Tax hikes, however, induce the private sector to invest less and to evade taxes by producing in the underground sector. Eventually, falling tax revenues force the government to either implement further tax hikes or to default. Our quantitative findings suggest that the underground economy fosters sovereign default risk and deepens debt crises.

Keywords: sovereign debt, default, fiscal policy, underground economy, tax evasion

JEL-Codes: E62, F34, H26

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1 Introduction

In the wake of sovereign debt crises, many countries have adopted fiscal consolidation policies in order to reduce public debt and to restore creditworthiness. However, particularly in crisis-prone countries, substantial underground activities undermine tax enforcement. Schneider, Buehn and Montenegro (2010) estimate that between 1999 and 2007 the average underground production measured in percent of official GDP amounts to 25.3% in Argentina, 27.5% in Greece, and 27.0% in Italy, compared to 16.0% in Germany, 8.6% in the United States, and 13.4% in all OECD countries.[1] The non-negligible size of the underground economy raises several important questions: How do tax evasion and the underground economy affect the government's policy choices between external borrowing, taxation, and public spending? How does limited tax enforcement impact sovereign default risk? How effective are fiscal consolidation policies in the presence of a large underground economy?

Motivated by these questions, this paper analyzes the dynamic interaction between sovereign debt, taxation, and the underground economy. We first explore the empirical relationship for a large sample of countries and find that the size of the underground economy is positively correlated with sovereign debt and interest spreads, particularly in the sample group of emerging economies.[2] We then rationalize these empirical regularities within a quantitative model of sovereign debt and default. We consider a small open economy in which a government finances a public good by taxing income and by issuing external long-term debt. The economy is divided into two sectors of production. In the official market sector, taxes are perfectly enforceable and production takes place using capital and labor. Production in the unofficial non-market (underground) sector uses labor only and is hidden from the tax authorities. Thus, households can evade taxes by running underground activities. Tax evasion, however, is illegal and bears the risk of getting detected and punished by the government. International financial markets are incomplete and debt contracts are not enforceable. In any period, the government has the option to default on its external debt. International creditors incorporate the probability of a sovereign default and charge a risk premium. In the event of a default, the government is temporarily excluded from international financial markets and finances public spending solely via taxation.

We study optimal fiscal policy in the presence of limited tax enforcement and default risk using the concept of Markov-perfect equilibria, in which the government moves first and the households form expectations about future public policies. In a quantitative exercise, we analyze the properties of optimal public policies and the private sector's responses and study the dynamics of a default event with a particular focus on the underground sector.

Our simulations reveal that the theoretical framework replicates the empirical regularities very

[1] The average in OECD countries is weigted by total GDP of each country in 2005.
[2] These findings are line with Elgin and Öztunalı (2012) and Elgin and Uras (2013).

well. In particular, our model predicts that the size of the underground economy is positively correlated with sovereign debt and interest spreads. We show that during debt crises, the dynamic interaction between sovereign default risk and the underground economy creates a "vicious circle": Higher sovereign risk premia tighten the endogenous borrowing constraint and force the government to raise taxes. Tax hikes, however, induce the private sector to invest less and to evade taxes by producing in the underground sector. In turn, falling tax revenues force the government to either implement further tax hikes or to default. Eventually, rasing taxes becomes too costly and the government finds it optimal to default.

After the default, once the government regains access to international financial markets, the vicious circle is reversed and becomes a "virtuous circle": After a default, the debt burden is low, and the government is able to implement a low tax rate encouraging formal production activities. In turn, a growing formal sector raises tax revenues allowing the government to reduce the tax rate even further. The tax cuts amplify the positive impact on formal labor and investment and foster the recovery of the economy.

Our paper builds on the large quantitative literature on sovereign debt and default, initiated by Eaton and Gersovitz (1981), Arellano (2008), and Aguiar and Gopinath (2006). Recent contributions in this area of research analyze the role of fiscal policy (e.g., Cuadra, Sanchez and Sapriza (2010), Kaas, Mellert and Scholl (2020)), capital and investment dynamics (Gordon and Guerron-Quintana (2018), Park (2017)), and the importance of long-term debt (Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Chatterjee and Eyigungor (2015)). Our paper is related to these studies as we study optimal fiscal policy in the presence of sovereign default risk allowing for endogenous production and investment dynamics. Moreover, to account for the high level of sovereign debt, we include long-term debt.

Our main contribution is to explicitly account for the dynamic interaction between the underground economy and sovereign default risk. Our modeling approach of the underground economy follows Busato and Chiarini (2004), Orsi, Raggia and Turino (2014), and Pappa, Sajedi and Vella (2015) who study the role of non-market production in two-sector general equilibrium models with a representative agent. These studies explore the impact of fiscal consolidations in the presence of underground economic activities, but abstract from sovereign default risk.

Our paper is related to a recent paper by Pappada and Zylberberg (2019) who study a model of sovereign default in which with a certain probability in each period entrepreneurs can choose between the adoption of a formal and an informal production technology. The authors show that fiscal consolidations may not unambiguously lower default risk, which is related the vicious cycle presented here.

While we consider a representative agent setup, there are recent attempts to introduce the classic tax evasion mechnanism of Allingham and Sandmo (1972) into dynamic macroeconomic settings with heterogeneous agents, see, e.g., Maffezzoli (2011), Di Nola, Kocharkov, Scholl and Tkhir (2018), and Kotsogiannis and Mateos-Planas (2019). Studies that focus on heterogeneous firms in the underground economy are, e.g., Ordonez (2014) and Antunes and Cavalcanti (2007). All these papers abstract from sovereign debt and default, which is the focus of our paper.

The paper is structured as follows. In Section 2, we present our empirical analysis of the relationship between the underground economy, sovereign debt, and default risk. In Section 3 we describe the model environment and define the recursive equilibrium. Section 4 explains the calibration strategy and presents the quantitative analysis of our theoretical framework. Section 5 concludes.

2 Empirical Facts

In this section, we empirically analyze the interaction between the size of the underground economy, sovereign debt, and default risk. The methodology is similar to Elgin and Uras (2013), albeit with two differences: First, we differentiate between OECD countries and emerging market economies, and, second, we adopt the Emerging Market Bond Index (EMBI) to measure the sovereign bond yield in emerging economies. We document two empirical facts. First, in emerging economies, the size of the informal sector is positively correlated with the government debt-to-GDP ratio. Second, the size of the informal sector is positively correlated with the government bond yield in emerging economies as well as in OECD economies. These findings are in line with Elgin and Uras (2013).

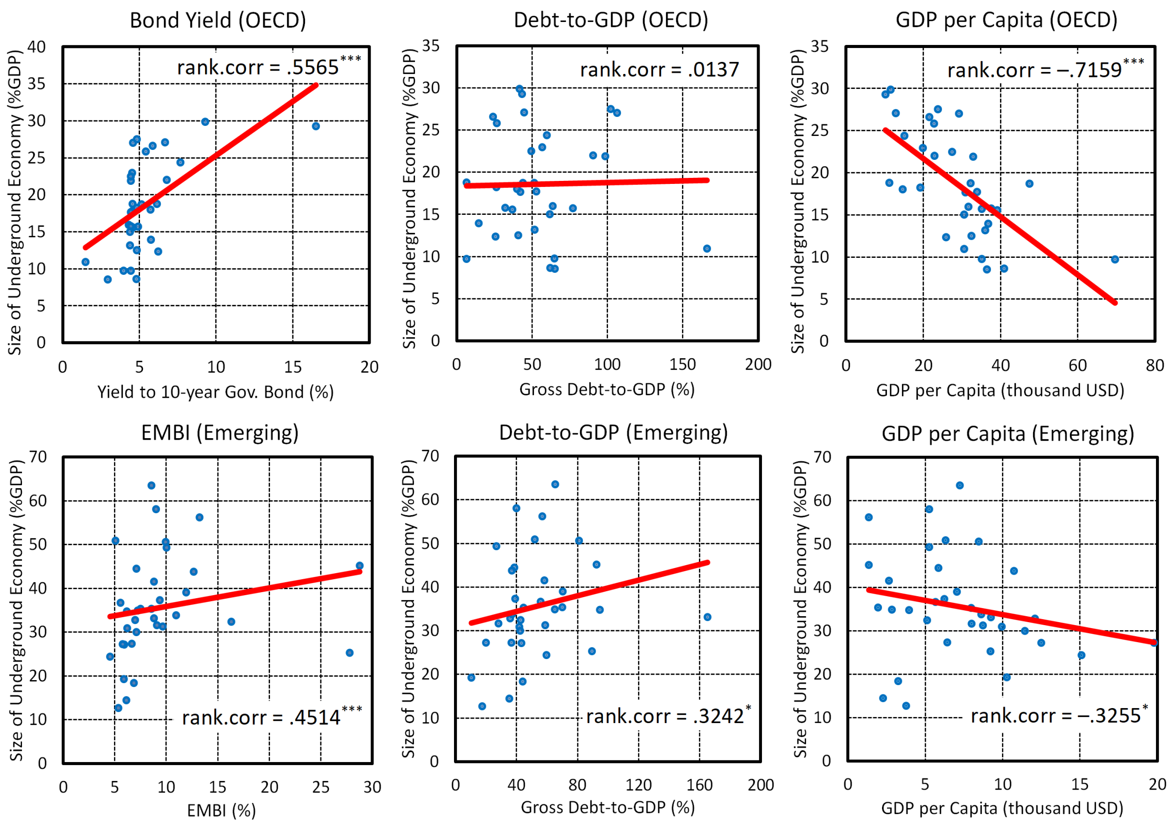
In a first step, Figure 1 illustrates how the size of the underground economy is related to the bond yield and to the debt-to-GDP ratio in the cross-section. For the sample of emerging economies, we measure the sovereign bond yield by the JPMorgan Emerging Market Bond Index (EMBI).[3] For the sample of OECD countries, we employ the 10-year government bond yield. As shown in the left column of the figure, countries with a larger informal sector tend to have a higher government bond yield. For both samples, the Spearman's rank correlations between the two variables are significant at the 1% level.[4] We find a significant positive correlation between the size of the informal sector and the debt-to-GDP ratio in emerging economies (middle column). For OECD countries, we do not find a significant interaction between the underground economy and sovereign debt.[5] As a supplement,

[3] Specifically, we use the stripped yield to maturity of the EMBI Global index. There are four versions of the EMBI index: EMBI, EMBI+, EMBI Global and the EMBI Global Diversified. The first index covers only the Brady bonds. The EMBI+ extends the coverage of the original EMBI to other emerging economies and to other dollar-denominated loans and Eurobonds. The EMBI Global extends the EMBI+ to cover additional countries and additional eligible instruments. The difference between the last two indices lies in the weighting of countries, which does not matter for our purposes. We use the EMBI Global as it provides the largest coverage.

[4] We calculate the rank correlation, instead of the Pearson's correlation, because the former is less affected by the outliers in our sample.

[5] In the scatterplot of the debt-to-GDP for OECD countries, the rightmost outlier is Japan. If we exclude Japan from the sample, the rank correlation rises to 0.0920, but remains insignficant.

Figure 1: Cross-Sectional Correlations



Notes: For the sample of OECD countries, the bond yield is measured by the 10-year government bond yield. For the sample of emerging economies the bond yield is measured by the stripped yield to maturity of the JP Morgan EMBI Global index. The red lines display best fits in the sense of least squares. "rank.corr" denotes the Spearman's rank correlation, whose significance at the 10%, 5%, and 1% level is indicated by *, ** and ***, respectively.

we plot the relationship between the informal sector and per capita GDP in the right column of Figure 1. We find that less wealthy countries tend to be characterized by larger underground economies.

In a second step, to capture the dynamics between the size of the underground economy, sovereign debt, and bond yields over time, we estimate one-way error component models with fixed individual effects:

$$\text{Indebtedness}_{i,t} = a + \alpha_i + \delta \text{SE}_{i,t} + \mathbf{x}_{i,t}^T \boldsymbol{\beta} + \varepsilon_{i,t}, \quad (1)$$

$$\text{Bond\_Yield}_{i,t} = a + \alpha_i + \delta \text{SE}_{i,t} + \mathbf{x}_{i,t}^T \boldsymbol{\beta} + \varepsilon_{i,t}. \quad (2)$$

$\text{Indebtedness}_{i,t}$ refers to the gross general government debt-to-GDP ratio. $\text{Bond\_Yield}_{i,t}$ denotes the sovereign bond yield, measured by the EMBI global in the sample of emerging economies, and by the 10-year government bond yield in the sample of OECD economies. $\text{SE}_{i,t}$ denotes the size of the underground economy. $\mathbf{x}_{i,t}$ refers to the vector of controls that could potentially explain the over-time variations of the dependent variables. Moreover, a is a constant, and α_i is an individual-specific intercept. Finally, i represents the individual country and t stands for time. ε_{it} is the error term.

Equations (1) and (2) are estimated separately for the sample of emerging economies and the sample of OECD countries. The sample of emerging economies consists of 34 countries while the sample of OECD economies contains 33 countries.[6] The panels are unbalanced. The data series are of annual frequency[7], and the time window is between 1999 and 2007. The size of the underground economy is taken from Schneider et al. (2010), who construct a comprehensive data set of the informal sector for 162 countries from 1999 to 2007.[8]

The control variables represent other factors that potentially affect the dependent variables. Per capita GDP, trade openness, and the GDP growth rate are taken from the Penn World Table 8.1. These variables are all adjusted by the purchasing power parity (PPP) to enable a cross-country comparison. The current-account balance and the annual rate of inflation are taken from the World Economic Outlook (WEO) data of the IMF. Moreover, we use the World Bank data for unemployment and tax revenues. To cover the political environment of each country, we specify six political indicators as independent variables: government stability, investment profile, corruption, law and order, democratic accountability and the bureaucratic quality. These indicators are taken from the International Country Risk Guide (ICRG).

Table 1 summarizes the descriptive statistics of the dependent and independent variables for both samples. Emerging economies are characterized by higher average bond yields and larger sizes of the underground economy. They tend to have higher inflation and unemployment, and lower scores on the political indicators. While the government debt-to-GDP ratios are similar in both samples, OECD countries have higher tax revenues. This implies that governments in emerging economies rely more on debt finance. One potential explanation for this phenomenon is that the larger informal sector limits the tax enforceability, which forces the governments to finance their expenditures via debt. Importantly, the sample of the emerging economies shows a larger hetereogeneity than the OECD countries, which can be seen in the high standard deviations for most variables.

Tables 2 and 3 report the regression results of equations (1) and (2). For each sample, we run four regressions to study the robustness of the results against the specification of the covariates. The first regression considers the size of the underground economy as the only independent variable while the second adds the statistics from the national accounts. The ICRG scores are taken into account in the third regression. The fourth regression applies all the above-mentioned independent variables plus tax revenues and the government-debt-to-GDP ratio.[9]

[6] See Table 7 in Appendix A for the list of countries included in each sample.

[7] To the best of our knowledge, there are only annual estimates of the size of the informal sector.

[8] Another wide-ranging data set of the informal sector is constructed by Elgin and Öztunalı (2012) using a model-based approach. Our empirical conclusions do not change when using their data set instead.

[9] Tax revenues are added only in the last regression because of missing values in the sample of the emerging economies. The government-debt-to-GDP ratio is used in regression (2).

Table 1: Summary Statistics

	OECD Countries					Emerging Economies				
Variable	Mean	Std.	Min.	Max.	Obs.	Mean	Std.	Min.	Max.	Obs.
Gov. Debt-to-GDP (%)	55.50	32.62	3.89	186.44	274	53.53	31.58	3.89	179.89	256
Bond Yield (%)	5.05	1.85	1.00	16.81	275	9.69	7.96	2.30	68.03	269
Under. Eco. (%GDP)	18.03	6.00	8.10	30.50	275	36.10	11.56	11.90	65.10	269
GDP *p.c.* (Thd. USD)	30.27	11.55	10.03	80.23	275	7.35	3.97	1.07	21.80	269
Trade Openness (%GDP)	85.85	49.46	21.20	314.03	275	72.82	38.22	21.20	214.35	269
Growth Rate (%)	3.86	3.89	-5.60	20.02	275	5.80	10.03	-60.34	75.24	269
Current Account (%GDP)	-0.10	6.36	-25.70	16.40	275	0.02	6.51	-25.24	25.34	269
Inflation (%)	2.66	1.85	-0.90	12.20	275	7.78	11.70	-1.40	96.10	269
Unemployment (%)	6.73	3.52	1.80	19.90	275	9.53	5.12	1.20	27.20	269
Tax Revenue (%GDP)	34.67	7.00	16.18	49.51	275	15.44	4.37	0.91	27.60	186
Gov. Stability	8.73	1.34	4.66	11.08	275	8.81	1.52	5.04	12.00	269
Investment Profile	10.94	1.28	6.00	12.00	275	8.13	2.13	2.00	12.00	269
Corruption	4.06	1.17	2.00	6.00	275	2.30	0.77	1.00	5.00	269
Law and Order	5.20	0.89	2.00	6.00	275	3.37	1.12	1.00	6.00	269
Demo. Accountability	5.74	0.46	4.00	6.00	275	4.10	1.47	0.00	6.00	269
Bureau. Quality	3.64	0.50	2.00	4.00	275	2.09	0.77	0.00	4.00	269

Notes: For the sample of OECD countries, the bond yield is measured by the 10-year government bond yield. For the sample of emerging economies the bond yield is measured by the stripped yield to maturity of the JP Morgan EMBI Global index. "Under. Eco" stands for the size of the underground economy, and "GDP *p.c.*" refers to per capita GDP. Moreover, "Std." denotes the standard deviation, and "Obs." refers the number of observations.

In emerging economies, we find a strong and robust positive correlation between the size of the informal sector and sovereign debt. In OECD countries, the debt-to-GDP ratio is largely explained by other covariates. Among other explanatory variables, the current account balance and the unemployment rate are positively correlated with sovereign debt. These findings are in line with Kim and Roubini (2008), who estimate a vector-autoregressive model and provide empirical evidence of a "twin divergence" between fiscal and current account deficits. Moreover, in emerging economies, per capita GDP is positively correlated with sovereign debt. In contrast, in OECD countries, such correlation is weak and negative. This finding reveals the procyclical feature of fiscal policies in emerging economies.

Table 2: Regression Results of Model (1)

	OECD Countries				Emerging Economies			
Ind.Variable	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Underground eco.	3.0430^{***}	-1.1967	-1.8921	-1.6131	5.0507^{***}	3.3801^{***}	4.1349^{***}	4.7708^{***}
	(0.9595)	(1.7107)	(1.8324)	(1.8017)	(.5484)	(.7720)	(.8015)	(.7234)
GDP per capita		$-.4903$	$-.6559^{*}$	$-.4862$		2.5224^{*}	3.6737^{**}	3.1169^{**}
		(.3564)	(.3867)	(.3838)		(1.4679)	(1.4882)	(1.2788)
Trade openness		$-.0123$	$-.0423$	$-.0667$		$-.2160^{*}$	$-.2496^{**}$	$-.1590$
		(.0744)	(.0783)	(.0773)		(.1162)	(.1132)	(.0975)
Growth rate		$-.0260$	.0549	.0575		$-.0340$	$-.0238$	$-.0430$
		(.1208)	(.1304)	(.1280)		(.1008)	(.0968)	(.0783)
Current account		$.4382^{**}$	$.4262^{**}$	$.6231^{***}$		$.3802^{*}$	$.3181^{*}$	.0292
		(.1913)	(.1972)	(.2040)		(.1958)	(.1920)	(.1739)
Inflation		.0941	.0707	$-.0791$		$.6412^{***}$	$.5077^{***}$	.1874
		(.3855)	(.3978)	(.3937)		(.1062)	(.1072)	(.1437)
Unemployment		1.4234^{***}	1.5333^{***}	1.5227^{***}		1.4260^{***}	1.3432^{**}	1.4985^{***}
		(.4210)	(.4267)	(.4191)		(.5360)	(.5169)	(.5150)
Gov. stability			.1762	.4374			-1.4844^{**}	-2.0304^{***}
			(.4860)	(.4848)			(.7338)	(.6970)
Investment Profile			$.9372^{*}$	1.1229^{**}			-1.3080^{*}	-1.2431
			(.5011)	(.4958)			(.7045)	(.8153)
Corruption			1.5828	1.0211			-1.7103	-1.8715
			(.9860)	(.9854)			(1.6030)	(1.4071)
Law and order			$-.3050$	$-.7489$			-4.6361^{***}	1.5880
			(1.3338)	(1.3178)			(1.4935)	(1.6342)
Demo. account.			$-.9184$	$-.8285$			.6743	.4028
			(1.9376)	(1.9030)			(1.3806)	(1.2319)
Bureau. quality			-3.8078	-3.8116			11.0161^{**}	-5.5127
			(3.0622)	(3.0072)			(4.4036)	(6.3873)
Tax revenue				1.3577^{***}				.0636
				(.4422)				(.5820)
(Intercept)	0.6586	83.3044^{**}	104.9189^{**}	50.0141	-128.2833^{***}	-88.8109^{***}	-102.9765^{***}	-110.4800^{**}
	(17.2990)	(36.9594)	(44.3838)	(47.1122)	(19.7585)	(33.0365)	(39.5834)	(42.8647)
Adjusted R^2	.9555	.9586	.9592	.9607	.8573	.8815	.8925	.9587
Observations	274	274	274	274	256	256	256	180

Notes: All regressions contain fixed country-specific effects. The numbers in parentheses are standard errors. Significance at the 10%, 5%, and 1% level is indicated by *, ** and ***, respectively.

Table 3: Regression Results of Model (2)

	OECD Countries				Emerging Economies			
Ind.Variable	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Underground eco.	1.1378***	.7295***	.4746***	.5894***	2.1344***	1.3240***	1.3424***	.6661**
	(.1046)	(.1729)	(.1782)	(.1707)	(.2281)	(.3371)	(.3485)	(.2674)
GDP per capita		−.0606*	−.0042	.0045		−.7737	−.5707	−.5007
		(.0362)	(.0378)	(.0364)		(.6263)	(.6282)	(.4203)
Trade openness		−.0123	−.0141*	−.0110		.0723	.0364	.0575*
		(.0075)	(.0076)	(.0073)		(.0509)	(.0492)	(.0316)
Growth rate		−.0159	−.0291**	−.0248**		−.0403	−.0264	−.0519**
		(.0122)	(.0127)	(.0121)		(.0415)	(.0397)	(.0252)
Current account		−.0017	.0138	.0038		−.0931	−.1760**	−.1230**
		(.0194)	(.0192)	(.0196)		(.0859)	(.0831)	(.0559)
Inflation		.2470***	.2295***	.2672***		.1662***	.1339***	.1394***
		(.0386)	(.0383)	(.0372)		(.0366)	(.0354)	(.0465)
Unemployment		−.0706*	−.0434	−.0618		.4350*	.4003*	.0258
		(.0427)	(.0417)	(.0407)		(.2285)	(.2190)	(.1707)
Gov. stability			.1182**	.1274***			−1.2888***	−.3295
			(.0475)	(.0459)			(.3009)	(.2311)
Investment Profile			−.1637***	−.1756***			−.8076***	−.0168
			(.0490)	(.0474)			(.2971)	(.2645)
Corruption			.1644*	.1031			1.2918*	1.1912**
			(.0958)	(.0934)			(.6683)	(.4556)
Law and order			−.1507	−.1483			−1.5003**	.9061*
			(.1307)	(.1247)			(.6510)	(.5276)
Demo. account.			−.2054	−.1941			.0255	−.8558**
			(.1898)	(.1801)			(.5842)	(.3965)
Bureau. quality			−.0144	−.0181			−1.3477	−.6276
			(.3001)	(.2855)			(1.8083)	(2.0606)
Tax revenue				−.0117				−.0416
				(.0427)				(.1872)
Debt-to-GDP				.0032				.0617**
				(.0062)				(.0275)
(Intercept)	−15.4698***	−5.3290	−.2550	−2.3800	−67.3578***	−42.8808***	−19.1872	−17.3892
	(1.8882)	(3.7389)	(4.3288)	(4.4685)	(8.2430)	(14.5112)	(16.7813)	(14.1239)
Adjusted R^2	.8341	.8671	.8784	.8911	.5843	.6248	.6661	.9296
Observations	275	275	275	274	269	269	269	180

Notes: All regressions contain fixed individual effects for each country/economy. The numbers in parentheses are standard errors. Significance at the 10%, 5%, and 1% level is indicated by *, ** and ***, respectively.

Table 3 highlights that the size of the underground economy is positively and significantly correlated with the sovereign bond yield in both samples. Importantly, such correlation is stronger in emerging economies than in OECD countries.[10]

3 The Model

We develop a small open economy in which there are three types of agents: households, international creditors, and a government. The government finances a public good by taxing income and by issuing external long-term debt. Following Busato and Chiarini (2004), Orsi et al. (2014), and Pappa et al. (2015), the economy is divided into two sectors of production. In the formal (or market) sector, production takes place using capital and labor. Production in the informal (or underground) sector uses labor only and is hidden to the tax authorities. Thus, households can evade taxes by running underground activities. Tax evasion, however, bears the risk of getting detected and punished by the government. As in Arellano (2008) and Aguiar and Gopinath (2006), international financial markets are incomplete and debt contracts are not enforceable. In any period, the government has the option to default on its external debt. International creditors incorporate the probability of a sovereign default and charge a risk premium. In the event of a default, the government is temporarily excluded from international financial markets and suffers an exogenous output loss.

3.1 The Environment

The small open economy is inhabited by an infinitely-lived representative household who produces and consumes a homogeneous consumption good. Her preferences are given by

$$U = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t), \tag{3}$$

where $\beta \in (0,1)$ denotes the rate of time preference. c_t and g_t refer to private consumption and government consumption, respectively. The utility function is differentiable, monotonically increasing and strictly concave in both arguments.

The economy is divided into two sectors of production: the formal (or market) sector and the informal (or underground) sector. In the market sector, the household employs capital and labor to produce the consumption good. Following Busato and Chiarini (2004, 2013), the production process of the underground sector uses labor only. In each period, the household is endowed with one unit of time, which she inelastically supplies as labor. Let $w_t \in (0,1)$ denote the share of labor time that the household allocates to market production, and $(1 - w_t)$ the share of labor time used for underground

[10] Note, however, that the sovereign bond yield is measured differently in the two samples. Among other things, the EMBI spread covers government debt instruments denominated in US dollars and Euros including bonds with all maturities.

production. The technologies of the market production $y_{m,t}$ and the underground production $y_{u,t}$ are given by:

$$y_{m,t} = e(z_t, d_t) k_t^{\theta} w_t^{1-\theta}, \tag{4}$$

$$y_{u,t} = e(z_t, d_t)(1 - w_t)^{1-\upsilon}, \tag{5}$$

where k_t denotes capital in period t. $0 < \theta < 1$ refers to the capital share in the official production sector. With $0 < \upsilon < 1$ the underground economy exhibits decreasing returns to scale, capturing the limited span of control as production icreases (Lucas, 1978). $z_t \in \mathcal{Z}$ denotes total factor productivity, which follows a Markov process with a Markov transition function $\mu(z_{t+1}, z_t)$. We assume that productivity is common to both production sectors. d_t is the sovereign's credit status in period t, which takes the value of 1 if the government defaults, or has previously defaulted and has not regained access to international financial markets. If the country is in a good credit standing, $d_t = 0$. The function $e(z_t, d_t)$ incorporates an output loss associated with a sovereign default: $e(z_t, d_t) = z_t$ if $d_t = 0$ and $e(z_t, d_t) \leq z_t$ if $d_t = 1$. We follow Busato and Chiarini (2004, 2013) and Orsi et al. (2014), and assume that the consumption good produced in the underground economy is indistinguishable from the one produced in the market sector. Therefore, in each period, we can normalize the common price of the consumption goods produced in two sectors to 1. Total production is thus given by

$$y_t = y_{m,t} + y_{u,t}.$$

The household owns the capital stock, makes production and investment decisions, and consumes. The household pays a tax τ_t on her income in the formal sector. As in Busato and Chiarini (2004), Orsi et al. (2014), and Pappa et al. (2015), she can hide her income from the tax authorities and evade taxes by producing in the underground economy. However, tax evasion is illegal and bears the risk of getting detected by the government. We follow Allingham and Sandmo (1972) and consider an exogenous probability $p \in (0, 1)$ of being audited. Once detected, the household is forced to pay the tax on the hidden income and is punished by an additional exogenous surcharge factor $s > 1$. The expected income from underground production can be stated as:

$$\mathrm{E}_t \mathcal{I}_{u,t} = (1 - p) y_{u,t} + p(1 - s\tau_t) y_{u,t} = (1 - ps\tau_t) y_{u,t}.$$

The household's budget constraint thus reads as:

$$c_t + k_{t+1} + \Theta(k_{t+1}, k_t) = (1 - \tau_t) y_{m,t} + (1 - ps\tau_t) y_{u,t} + (1 - \delta) k_t, \tag{6}$$

where δ is the capital depreciation rate and $\Theta(k_{t+1}, k_t)$ denotes convex capital adjustment costs.

The government is benevolent and finances government consumption g_t. In addition to raising an income tax, at period t, the government can purchase non-contingent long-term bonds discounted at price $q(b_{t+1}, k_{t+1}, z_t)$ in international financial markets. $q(\cdot)$ represents the bond-price schedule, which the sovereign takes as given, and $b_t < 0$ denotes the outstanding bond stock held by the government at period t.[11] We adopt the approach by Chatterjee and Eyigungor (2012) and let each unit of outstanding bond b_t mature with probability λ at period t. If a bond unit does not mature, the sovereign has to pay a coupon $n > 0$ to the creditor. The new bond purchase at period t is given as $b_{t+1} - (1 - \lambda)b_t$. If the government fulfills the current debt obligations, its budget constraint writes as

$$g_t = \tau_t y_{r,t} + ps\tau_t y_{u,t} + \big(\lambda + (1-\lambda)n\big)b_t - q(b_{t+1}, k_{t+1}, z_t)\big(b_{t+1} - (1-\lambda)b_t\big). \tag{7}$$

The term $\big(\lambda + (1 - \lambda)n\big)b_t$ captures the sum of the debt-repayments related to the fraction λ that matures and the coupon-payments related to the remaining fraction $(1 - \lambda)$. The term $q(b_{t+1}, k_{t+1}, z_t)\big(b_{t+1} - (1 - \lambda)b_t\big)$ is the value of the period-$(t + 1)$ outstanding bond at the current bond price.

Debt contracts are not enforceable and subject to default risk. In each period, contingent on having access to international financial markets, the government decides whether to repay or to default on its outstanding debt. Sovereign default has four consequences. First, all existing debt and coupon obligations are written off. Second, the sovereign is immediately excluded from international financial markets and enters financial autarky. During financial autarky, the government can only rely on the income tax to finance government consumption. Third, starting from the next period after the default, the sovereign faces an exogenous probability ϕ to re-enter international financial markets. Fourth, as described above, financial autarky is associated with an exogenous output loss, $e(z_t, 1) \leq z_t$. If the government defaults at period t and/or the sovereign is in financial autarky, the government's budget constraint is

$$g_t = \tau_t y_{r,t} + ps\tau_t y_{u,t}. \tag{8}$$

International creditors provide long-term debt contracts to the government. They are risk-neutral, perfectly competitive and have perfect information about the sovereign's past and current debt level, the capital stock, and the productivity realizations. Moreover, international creditors can borrow or lend from international financial markets at the constant risk-free interest rate r_f.

3.2 The Recursive Equilibrium

In the following, we consider the government-moves-first Markov-perfect equilibrium as defined by Ortigueira (2006). Specifically, in period t, the government takes instantaneous leadership and chooses its fiscal policies before the household decides on consumption, investment, and pro-

[11] We follow the convention of the literature and model borrowing as negative bond holdings.

duction. Thus, the government incorporates the private sector's response when making its optimal choices on taxation, debt issuance, and default.

3.2.1 Private Sector

The current state of the economy can be characterized by (b, k, z). In each period, the household chooses c, k' and w to maximize her expected lifetime utility (3), subject to the constraints (4), (5) and (6). The household takes as given the government's fiscal policies d, g, τ and b', and expects the government to follow the policy rules $d' = \mathcal{D}(b', k', z')$, $g' = \mathcal{G}(b', k', z')$, $\tau' = \mathcal{T}(b', k', z')$ and $b'' = \mathcal{B}(b', k', z')$ in the future period. Let $\mathbb{W}\big(b, k, z \,\big|\, d, g, \tau, b'\big)$ denote the household's (conditional) value function. If the current default decision is $d = 0$, the economy has access to international financial markets and $\mathbb{W}(\cdot)$ can be recursively defined as follows:

$$\begin{aligned} \mathbb{W}\big(b, k, z \,\big|\, 0, g, \tau, b'\big) = \max_{\{c, k', w\}} \Big\{ & u(c, g) + \\ & \beta \int_{z'} \mathbb{W}\Big(b', k', z' \,\Big|\, \mathcal{D}(b', k', z'), \mathcal{G}(b', k', z'), \mathcal{T}(b', k', z'), \mathcal{B}(b', k', z')\Big) \mu\big(z', z\big)\, \mathrm{d}z' \Big\} \end{aligned} \tag{9}$$

subject to

$$c + k' + \Theta(k', k) = (1-\tau)y_m + (1 - ps\tau)y_u + (1-\delta)k,$$

$$y_m = e(z, 0)k^{\theta} w^{1-\theta},$$

$$y_u = e(z, 0)(1-w)^{1-\upsilon}.$$

If the current default decision is $d = 1$, then $b' = 0$, and the sovereign loses access to international financial markets. Starting from the next period, the sovereign has an exogenous probability ϕ in each period to re-enter international financial markets. The economy suffers from the output loss during financial autarky. In this case, $\mathbb{W}(\cdot)$ can be recursively defined as follows:

$$\begin{aligned} \mathbb{W}\big(b, k, z \,\big|\, 1, g, \tau, 0\big) = \max_{\{c, k', w\}} \Big\{ & u(c, g) + \\ & \beta \int_{z'} \Big[\phi \mathbb{W}\Big(0, k', z' \,\Big|\, \mathcal{D}(0, k', z'), \mathcal{G}(0, k', z'), \mathcal{T}(0, k', z'), \mathcal{B}(0, k', z')\Big) + \\ & (1-\phi)\mathbb{W}\Big(0, k', z' \,\Big|\, 1, \mathcal{G}(0, k', z'), \mathcal{T}(0, k', z'), 0\Big)\Big] \mu\big(z', z\big)\, \mathrm{d}z' \Big\} \end{aligned} \tag{10}$$

subject to

$$c + k' + \Theta(k', k) = (1-\tau)y_m + (1 - ps\tau)y_u + (1-\delta)k,$$

$$y_m = e(z, 1)k^{\theta} w^{1-\theta},$$

$$y_u = e(z, 1)(1-w)^{1-\upsilon}.$$

In the following, we denote the solution to the constrained maximization problem (9) or (10) as $c = \mathcal{C}(b, k, z \,|\, d, g, \tau, b')$, $k' = \mathcal{K}(b, k, z \,|\, d, g, \tau, b')$ and $w = \mathcal{W}(b, k, z \,|\, d, g, \tau, b')$, for $d = 0, 1$.

3.2.2 Public Sector

The benevolent government takes into account the optimal responses of the household $\mathcal{C}(\cdot)$, $\mathcal{K}(\cdot)$ and $\mathcal{W}(\cdot)$, and maximizes the household's expected lifetime utility (3) by choosing fiscal policies d, g, τ and b' subject to the constraints (4), (5), (7) and (8). Conditional on having access to international financial markets, the government has the option to fulfill the external debt obligations or to default:

$$\mathbb{V}(b,k,z) = \max\left\{\mathbb{V}^r(b,k,z)\ ,\ \mathbb{V}^d(k,z)\right\}. \tag{11}$$

where $\mathbb{V}^r(b,k,z)$ denotes the value function associated with debt repayment, and $\mathbb{V}^d(k,z)$ is the value function under financial autarky.

If the government fulfills its current debt obligations, $d=0$, it can issue new debt. Taking the bond price schedule $q(b',k',z)$ as given, the government solves

$$\mathbb{V}^r(b,k,z) = \max_{\{g,\tau,b'\}}\left\{u(c,g)+\beta\int_{z'}\mathbb{V}(b',k',z')\mu(z',z)dz'\right\} \tag{12}$$

subject to

$$\begin{aligned}
&g = \tau y_m + ps\tau y_u + \big(\lambda+(1-\lambda)n\big)b - q(b',k',z)\big(b'-(1-\lambda)b\big),\\
&y_m = e(z,0)k^{\theta}w^{1-\theta},\\
&y_u = e(z,0)(1-w)^{1-\upsilon},\\
&k' = \mathcal{K}(b,k,z|0,g,\tau,b'),\quad c=\mathcal{C}(b,k,z|0,g,\tau,b'),\ \text{and}\ \ w=\mathcal{W}(b,k,z|0,g,\tau,b').
\end{aligned}$$

If the government defaults, $d=1$, the existing debt and coupon obligations are written off, and the government is in financial autarky, during which the productivity is reduced to $e(z,1)\leq z$. Taking as given the probability ϕ of regaining access to international financial markets, the government solves

$$\mathbb{V}^d(k,z) = \max_{\{g,\tau\}}\left\{u(c,g)+\beta\int_{z'}\Big[\phi\mathbb{V}(0,k',z')+(1-\phi)\mathbb{V}^d(k',z')\Big]\mu(z',z)dz'\right\} \tag{13}$$

subject to

$$\begin{aligned}
&g = \tau y_m + ps\tau y_u,\\
&y_m = e(z,1)k^{\theta}w^{1-\theta},\\
&y_u = e(z,1)(1-w)^{1-\upsilon},\\
&k' = \mathcal{K}(b,k,z|1,g,\tau,0),\quad c=\mathcal{C}(b,k,z|1,g,\tau,0),\ \text{and}\ \ w=\mathcal{W}(b,k,z|1,g,\tau,0).
\end{aligned}$$

$d = 1$ if the sovereign has defaulted in the past and has not yet regained access to international financial markets. Otherwise, the government determines d based on (11):

$$d = \mathcal{D}(b, k, z) = \begin{cases} 1 & \text{if } \mathbb{V}^r(b, k, z) < \mathbb{V}^d(k, z), \\ 0 & \text{else.} \end{cases} \tag{14}$$

By solving the constrained optimization problems (11), (12) and (13), the government obtains its optimal fiscal policies $g = \mathcal{G}(b, k, z)$, $\tau = \mathcal{T}(b, k, z)$ and $b' = \mathcal{B}(b, k, z)$.

3.2.3 Bond Price

Following Arellano (2008), the world risk-free interest rate r_f is exogenous. Moreover, since risk-neutral international creditors are perfectly competitive, the zero-expected-profit condition holds such that

$$q(b', k', z) = \int_{z'} \left[1 - \mathcal{D}(b', k', z')\right] \frac{\lambda + (1 - \lambda)\left[n + q(b'', k'', z')\right]}{1 + r^f} \mu(z', z) \,\mathrm{d}z'. \tag{15}$$

The bond price function (15) is the same as in Chatterjee and Eyigungor (2012) and Gordon and Guerron-Quintana (2018). The creditor weights $q(b', k', z)$ against the discounted future benefit (right-hand side of equation (15)) of owning a bond unit. In the event of a default in the next period, $\mathcal{D}(b', k', z') = 1$, the creditor gets nothing. If the sovereign does not default, the creditor receives the fraction λ of the bond that matures. For the fraction $(1 - \lambda)$ that does not mature, the creditor receives the coupon payment n as well as the market value $q(b'', k'', z')$.

3.2.4 Equilibrium Definition

The recursive equilibrium for the small open economy is defined as a set of value functions $\mathbb{V}(b, k, z)$, $\mathbb{V}^r(b, k, z)$, $\mathbb{V}^d(k, z)$, $\mathbb{W}(b, k, z \,|\, d, g, \tau, b')$, a set of (conditional) policy functions for household's consumption $\mathcal{C}(b, k, z \,|\, d, g, \tau, b')$, next-period capital stock $\mathcal{K}(b, k, z \,|\, d, g, \tau, b')$ and official labor share $\mathcal{W}(b, k, z \,|\, d, g, \tau, b')$, a set of policy functions for the sovereign's default decision $\mathcal{D}(b, k, z)$, government consumption $\mathcal{G}(b, k, z)$, taxation $\mathcal{T}(b, k, z)$, the next-period debt $\mathcal{B}(b, k, z)$, and a bond price function $q(b', k', z)$ such that

1. given the government's fiscal policies d, g, τ and b', the household's policy and value functions $\mathcal{C}(b, k, z \,|\, d, g, \tau, b')$, $\mathcal{K}(b, k, z \,|\, d, g, \tau, b')$, $\mathcal{W}(b, k, z \,|\, d, g, \tau, b')$ solve the constrained maximization problems (9) and (10) ;
2. given the bond price function $q(b', k', z)$ and the household's policy and value functions $\mathcal{C}(b, k, z \,|\, d, g, \tau, b')$, $\mathcal{K}(b, k, z \,|\, d, g, \tau, b')$ and $\mathcal{W}(b, k, z \,|\, d, g, \tau, b')$, the government's policy functions $\mathcal{G}(b, k, z)$, $\mathcal{T}(b, k, z)$, $\mathcal{B}(b, k, z)$ and the default decision $\mathcal{D}(b, k, z)$ solve (11), (12), (13) and (14);

3. bond prices $q(b', k', z)$ fulfill equation (15) such that risk-neutral international creditors earn zero expected profits.

4 Quantitative Analysis

4.1 Data

In our quantitative analysis, we apply the model to the Argentine economy to study the dynamic interaction of sovereign default risk, fiscal policy, and the underground economy. Table 6 summarizes selected empirical facts for the Argentine economy. For the business cycle statistics, we exclude the sovereign default event of 2002 and consider quarterly data from 1994Q1 to 2001Q4.[12] The estimated size of the underground economy is taken from Elgin and Öztunalı (2012).[13] The time series for real private consumption, real GDP, real investment, and net exports are from the National Institute of Statistics and Censuses (INDEC) of Argentina. The sovereign interest spread is measured by the JP Morgan Emerging Markets Bond Index (EMBI Global) for Argentina. Net exports are given as a percentage of output. The series are linearly detrended.

The empirical statistics highlight that the Argentine economy is substantially indebted, facing a mean sovereign spread of 7.7%. The size of the underground economy is non-negligible and amounts to 23$\%$ of national income. Underground activities are countercyclical. Importantly, the size of the underground economy is positively correlated with the sovereign spread, as discussed in Section 2. Similar cyclical properties of the underground economy are reported by Busato and Chiarini (2004). In line with previous studies, e.g., Arellano (2008), Neumeyer and Perri (2005), output is negatively correlated with the sovereign spread, private consumption is more volatile than output, and net exports are countercyclical. Investment is 2.5 times more volatile than output and strongly procyclical.

4.2 Measuring Official Output in the Data and in the Model

Does the official GDP, which is reported in the national accounts, already include the underground activities? This question is surprisingly hard to answer. Our data series of GDP are based on the System of National Accounts (SNA) 2008, which recommends to include the underground economy in the national accounts. However, it is not straightforward to know whether this recommendation is put into practice by the national statistics authority. There are different opions and treatments in the literature. Let $y_{o,t}$ denote the official measure of GDP. Busato and Chiarini (2004,

[12] Data sources and descriptions are summarized in Appendix B.

[13] Although the data from Schneider et al. (2010) are used in our empirical analysis, the data only date back to the year 1999. Elgin and Öztunalı (2012), on the other hand, provide data from 1950 to 2009. Recently, there has been a new dataset from Medina and Schneider (2018), which covers 158 countries from 1991 to 2015. At least for Argentina, these three datasets have similar first moments of the size of the underground economy. Figure 7 compares the three datasets around the Argentina's default event in 2002.

Table 4: Model Generated Properties of Formal/Informal Consumption and Output

Variable (X)	$y_{u,t}$	$y_{m,t}$	$y_{o,t}$	y_t	$c_{m,t}$	$c_{o,t}$	c_t
$\text{E}(X)$	0.38	1.64	1.65	2.02	1.12	1.13	1.50
σ_X	4.01	4.85	4.82	4.09	5.37	5.30	3.85
$\rho(X, y_{m,t})$	0.12	—	0.99	0.98	0.87	0.88	0.95
$\rho(X, y_{o,t})$	0.13		—	0.98	0.87	0.87	0.95
$\rho(X, y_t)$	0.29			—	0.78	0.78	0.89

Notes: $\text{E}(\cdot)$, $\sigma_\cdot$ and $\rho(\cdot,\cdot)$ denote the mean, standard deviation and correlation, respectively.

2013) and Orsi et al. (2014) define the official GDP in Italy as $y_{o,t} := y_{m,t} + y_{u,t}$. On the other hand, when estimating the size of underground sector in Latin America, Solis-Garcia and Xie (2018) match official GDP as $y_{o,t} := y_{m,t}$. The latter definition is also adopted by Elgin and Öztunalı (2012).

As we calibrate our model to Argentina, we follow Solis-Garcia and Xie (2018) and assume that the offical GDP measure is not all-inclusive. But different from Solis-Garcia and Xie (2018), we assume that with probability p the underground activities are detected and enter the reported official measure of GDP:

$$y_{o,t} = y_{m,t} + p y_{u,t}. \tag{16}$$

Let $i_t := k' - (1-\delta)k + \Theta(k', k)$ represent the investment of the private sector, and nx_t denote net exports. In our model, the national account identity is given by

$$c_t + g_t + i_t + nx_t = y_{m,t} + y_{u,t}.$$

To maintain the national account identity, we define the officially measured private consumption expenditures $c_{o,t}$ as

$$\underbrace{\left[c_t - (1-p)y_{u,t}\right]}_{=:c_{o,t}} + g_t + i_t + nx_t = \underbrace{y_{m,t} + p y_{u,t}}_{=:y_{o,t}}.$$

Note that this specification relies on the assumption that underground output is consumed rather than invested.

To explore the sensitivity of these measures, in Table 4, we report the model-generated statistical properties of $c_{o,t}$ and $y_{o,t}$ together with $c_{m,t}$, $y_{m,t}$, $c_{u,t}$, and $y_{u,t}$. As can be seen from the table, comparing to $c_{m,t}$ and $c_{o,t}$, c_t has a lower volatility but a higher correlation with production. The volatility and correlation of y_t, on the other hand, are close to that of $y_{m,t}$ and $y_{o,t}$. Most importantly, $y_{o,t}$ and $c_{o,t}$ have very similar statistical properties as $y_{m,t}$ and $c_{m,t}$. This is mainly because p is very small in our calibration. Thus, the quantitative implications of our model are similar if we define the official GDP as $y_{m,t}$ instead.

4.3 Functional Forms and Calibration

To calibrate the model to the Argentine economy, we specify functional forms and choose parameter values on a quarterly basis. Table 5 summarizes the set of parameters and indicates whether the parameter values are chosen directly or calibrated to match empirical targets.

Following Busato and Chiarini (2004, 2013) and Cuadra et al. (2010) utility takes the conventional weighted-CRRA form:

$$u(c_t, g_t) = \frac{c_t^{1-\eta} - 1}{1-\eta} + \alpha \frac{g_t^{1-\eta} - 1}{1-\eta}, \tag{17}$$

where $\eta > 0$, $\eta \neq 1$ denotes the parameter of relative risk aversion and $\alpha > 0$ is a preference weight. We follow the macroeconomic literature and choose $\eta = 2$. α is calibrated to match government consumption as a percentage share of official GDP (12%). The rate of time preference β is chosen to replicate the debt-to-GDP ratio. According to Chatterjee and Eyigungor (2012), for the period from 1993Q1 to 2001Q4, 70% of the external Argentine debt was defaultable. Our model generates an average debt share of 104.26%, which corresponds to 72% defaultable debt.

As in Park (2017), capital adjustment costs are given as:

$$\Theta(k_{t+1}, k_t) = \frac{\kappa}{2}\left(1 - \frac{k_{t+1}}{k_t}\right)^2 k_t. \tag{18}$$

κ is chosen to match the volatility of investment in Argentina, which is 2.56 times higher than the volatility of official GDP. Capital depreciation δ is chosen to replicate the average ratio of investment to official GDP, which is 17% in Argentina.

We calibrate the parameter υ of the production function in the informal sector to match the empirically observed share of underground labor.

Following the convention in the business-cycle literature, the logarithm of z_t is assumed to follow an AR(1) process:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_t, \tag{19}$$

with ε_t being i.i.d. $N(0, \sigma_\varepsilon^2)$. We follow Neumeyer and Perri (2005) and set the persistence ρ_z equal to 0.95. σ is calibrated to replicate the volatility of official GDP in Argentina. Productivity is assumed to be given by:

$$e(z_t, d_t) = (1 - d_t)z_t + d_t(1 - \chi(z_t))z_t.$$

If the government does not default $d_t = 0$, productivity is given by $e(z_t, 0) = z_t$. If the government chooses to default, $d_t = 1$, productivity takes the value $e(z_t, 1) = (1 - \chi(z_t))z_t$, where $\chi(z_t)$ denotes the asymmetric output cost as in Gordon and Guerron-Quintana (2018) and Chatterjee and Eyigungor (2012):

$$\chi(z_t) = \min\left\{ \max\left\{\chi_0 + \chi_1 z_t\, ,\, 0\right\} , 1\right\}.$$

χ_1 and χ_2 are set to match the average sovereign spread of 7.70% and the volatility of the spread of 3.51%. Following Chatterjee and Eyigungor (2015), we impose an upper bound ν on the default probability in order to prevent debt dilution.

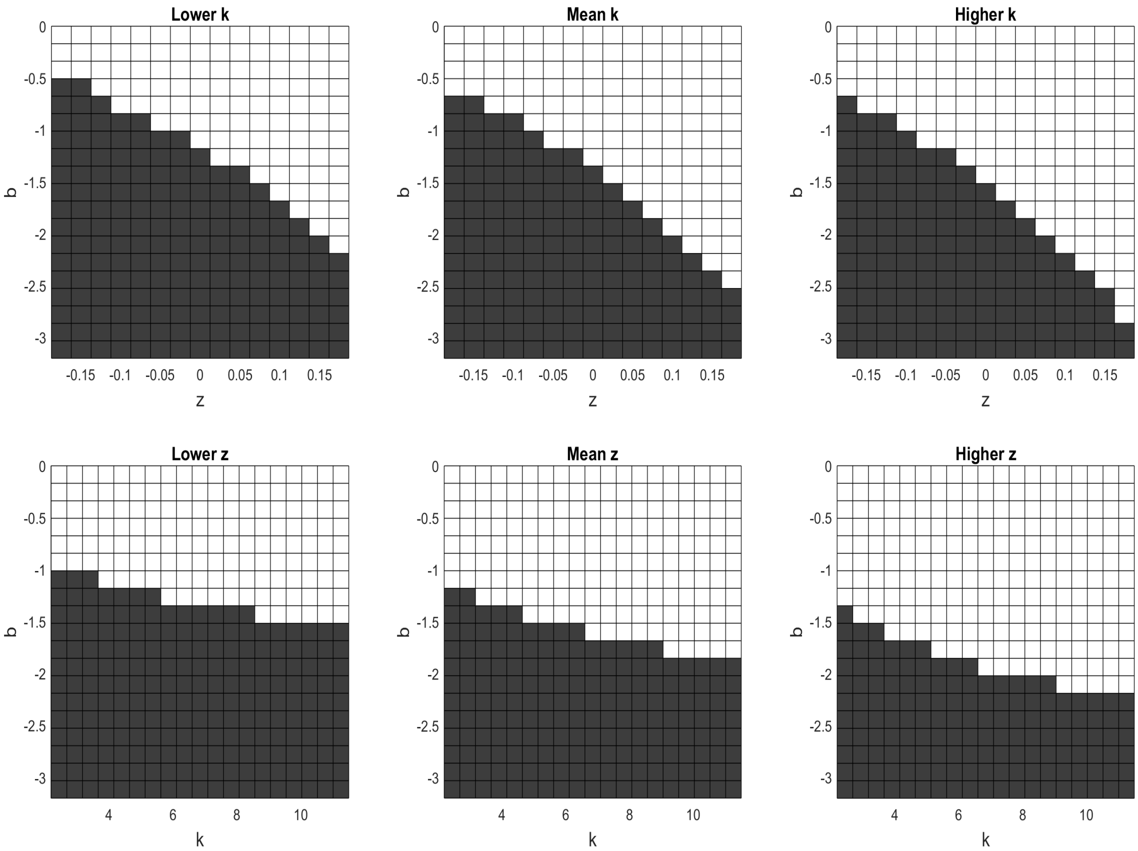
In the theoretical model, two parameters affect the size of the underground economy: the probability p that tax evasion is detected by the government, and the penalty s to be paid by the household on the hidden income once detected. In the calibration exercise, these two parameters are not distinguishable.[14] Therefore, we choose the expected penalty ps. We calibrate $ps = 0.06$ as to match the empirically observed size of the underground economy of 23% in Argentina. Considering Italy, Orsi et al. (2014) and Busato and Chiarini (2004) use a detection probability of $p = 0.03$ and a surcharge factor $s = 1.3$, implying an expected penalty $ps = 0.0309$, which is slightly lower than our value.

[14] Strictly speaking, given our definition of official GDP in equation (16), p matters for the informal sector's share in GDP. However, it is confirmed by, e.g., Busato and Chiarini (2004, 2013), Orsi et al. (2014), and Solis-Garcia and Xie (2018) that the detection probability p is very small, usually close to 0. We thus set p to be a small number ($p = 0.03$) and calibrate the expected tax evasion penalty ps.

Table 5: Calibration

Parameter		Value	Target/Source
r	the risk-free interest rate	0.01	(standard value)
β	time discount factor	0.951	ratio of defaultable debt to official GDP
η	relative risk aversion	2.00	(standard value)
α	weight on gov'ment consumption	0.025	ratio of gov. consumption to official GDP
ps	expected surcharge on tax evasion	0.06	size of the underground economy
δ	capital depreciation	0.0555	ratio of investment to official GDP
κ	capital adjustment cost	5.50	volatility of investment
θ	capital share in market production	0.39	standard value
υ	parameter in underground production	0.36	share of underground labor
σ	volatility of productivity shock	0.0194	volatility of official GDP
ρ	persistence of productivity shock	0.95	Neumeyer and Perri (2005)
λ	probability of maturity	0.05	Chatterjee and Eyigungor (2012)
n	coupon payment	0.03	Chatterjee and Eyigungor (2012)
ϕ	probability of re-entry after default	0.10	Chatterjee and Eyigungor (2012)
χ_0	intercept of output cost	-0.2488	mean of sovereign spread
χ_1	slope of output cost	0.3458	volatility of sovereign spread
ν	upper bound of default probability	0.75	Chatterjee and Eyigungor (2015)

Figure 2: Default Decision



Notes: This figure displays the default region (grey area). Mean k refers to average capital during non-default episodes in the simulation. Higher and lower k are $\pm 25\%$ of the mean of k, respectively. Mean z is the mean productivity during non-default episodes in the simulation. Higher and lower z are $\pm 5\%$ of the mean of z, respectively.

4.4 Quantitative Results

4.4.1 Default Regions and Bond Price Schedule

The model has standard implications with respect to the default region and the bond price schedule, which we briefly discuss in this subsection. Figure 2 displays the optimal decision of the government whether to repay or to default on its outstanding debt obligations. In the first row of the figure, we fix capital k at three different values and plot the default decision $\mathcal{D}(b, k, z)$ as a function of bonds b and the productivity realization z. The second row fixes z at different levels, and plots $\mathcal{D}(b, k, z)$ against b and k. The grey area refers to the states in which the government finds it optimal to default. Clearly, the default area is smaller for better productivity realizations, for higher capital stocks, and for lower levels of debt.

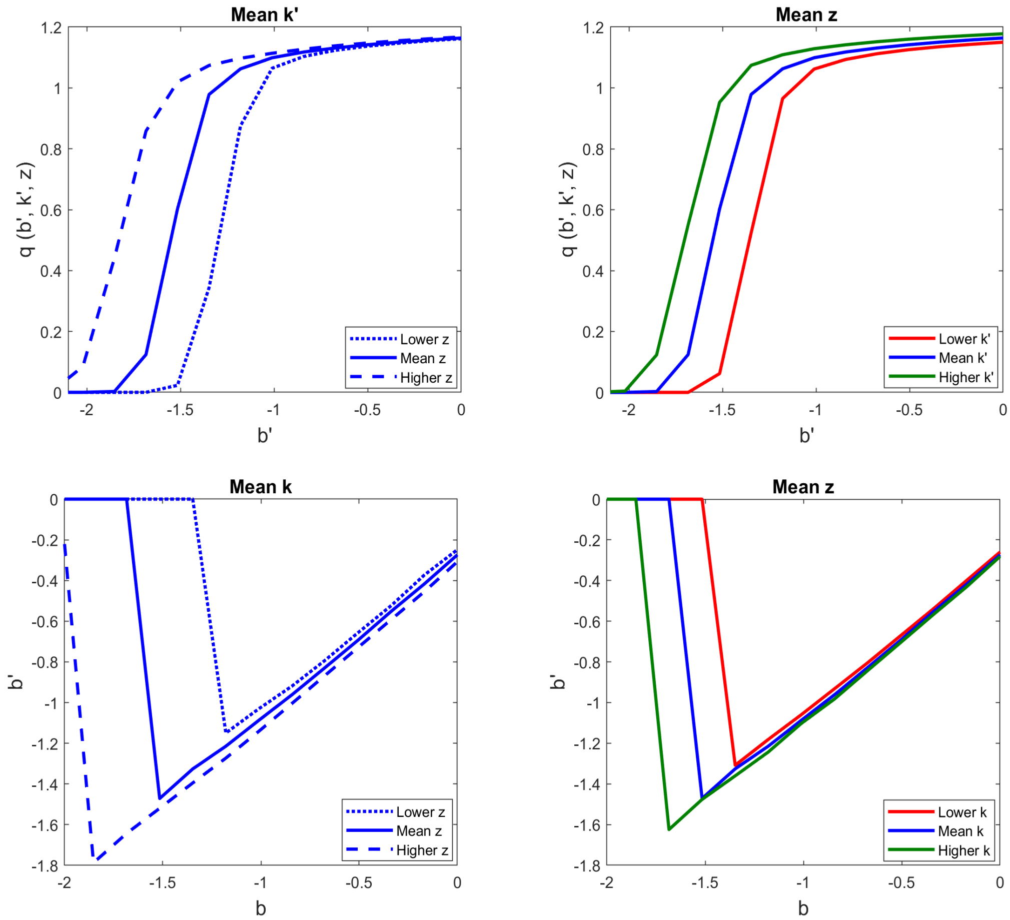
The first row of Figure 3 shows the sovereign bond price $q(b', k', z)$, keeping k' and z fixed at their respective means of the model-simulated non-default episodes. In the second row, we plot the government's borrowing policy $\mathcal{B}(b, k, z)$. It is evident that, first, the bond price is increasing in b (decreasing in the sovereign's debt burden). With the accumulation of debt, international creditors incorporate the rising sovereign default risk in their pricing decision and charge a larger premium on sovereign debt. Second, the bond price decreases if the economy faces adverse productivity realizations. In times of low productivity, the government is less able to fulfill its debt obligations and the higher sovereign default risk is reflected in the bond price. Third, the bond price is increasing in capital. If the economy operates with a higher capital stock, the larger production raises tax revenues and reduces the probability of a sovereign default. In summary, in times of high sovereign debt and adverse productivity realizations, the government becomes borrowing-constrained due to high sovereign risk premia. In contrast, higher capital reduces the credit costs and, thus, relaxes the endogenous borrowing constraint.

4.4.2 The Interaction between Sovereign Debt, Taxation, and the Underground Economy

Figure 4 displays the tax policy function. The tax rate is increasing in sovereign debt and decreasing in productivity. Except for case of low debt, in which the government is not credit-constrained, $\mathcal{T}(\cdot)$ is decreasing in capital k.[15] This pattern implies that fiscal policy is procyclical: In times of low productivity/low capital or high debt, the government becomes borrowing-constrained and has to increase the tax rate to finance government consumption. In default, however, debt is not repaid, which allows the government to reduce the tax rate. These findings correspond to those reported in previous studies, e.g., Cuadra et al. (2010), Kaas et al. (2020), Fink and Scholl (2016), and

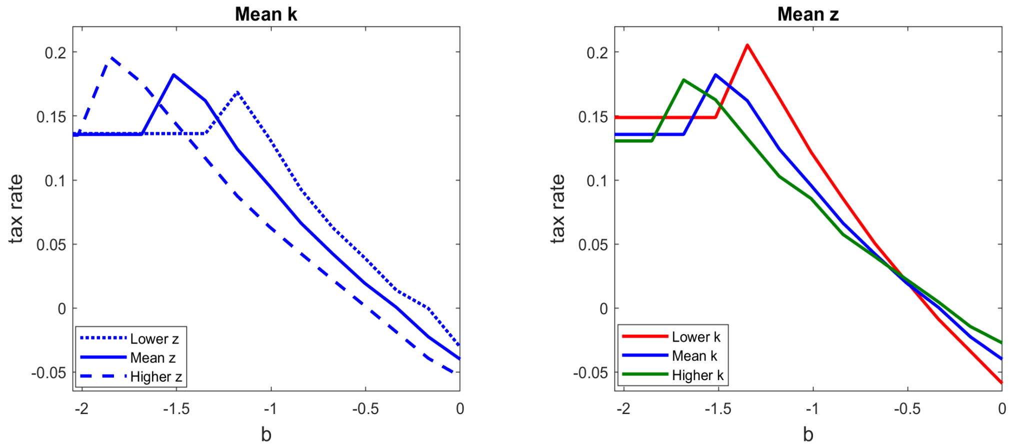
[15] The credit constraint is much looser in the model with long-term debt. A sovereign with a low debt burden has the fiscal space to issue debt. If the capital stock is low and the marginal cost of taxation on the private sector is high, the sovereign has an incentive to reduce the tax rate and to increase borrowing. In the extreme case, the sovereign imposes a negative tax rate to subsidize the private sector, as suggested by the figure.

Figure 3: Bond Price and Borrowing Policy



Notes: This figure displays the bond price function $q(b', k', z)$ and the borrowing policy $\mathcal{B}(b, k, z)$. The panels in the left column display $q(b', k', z)$ and $\mathcal{B}(b, k, z)$ as a function of b' (b) evaluated at the average value of capital considering different productivity realizations. Higher and lower z are $\pm 5\%$ of the mean of z. The panels in the right column display $q(b', k', z)$ and $b'(b, k, z)$ as a function of b' (b) evaluated at the average value of productivity considering different values of capital. Higher and lower k are $\pm 25\%$ of the mean of k, respectively.

Figure 4: Tax Policy



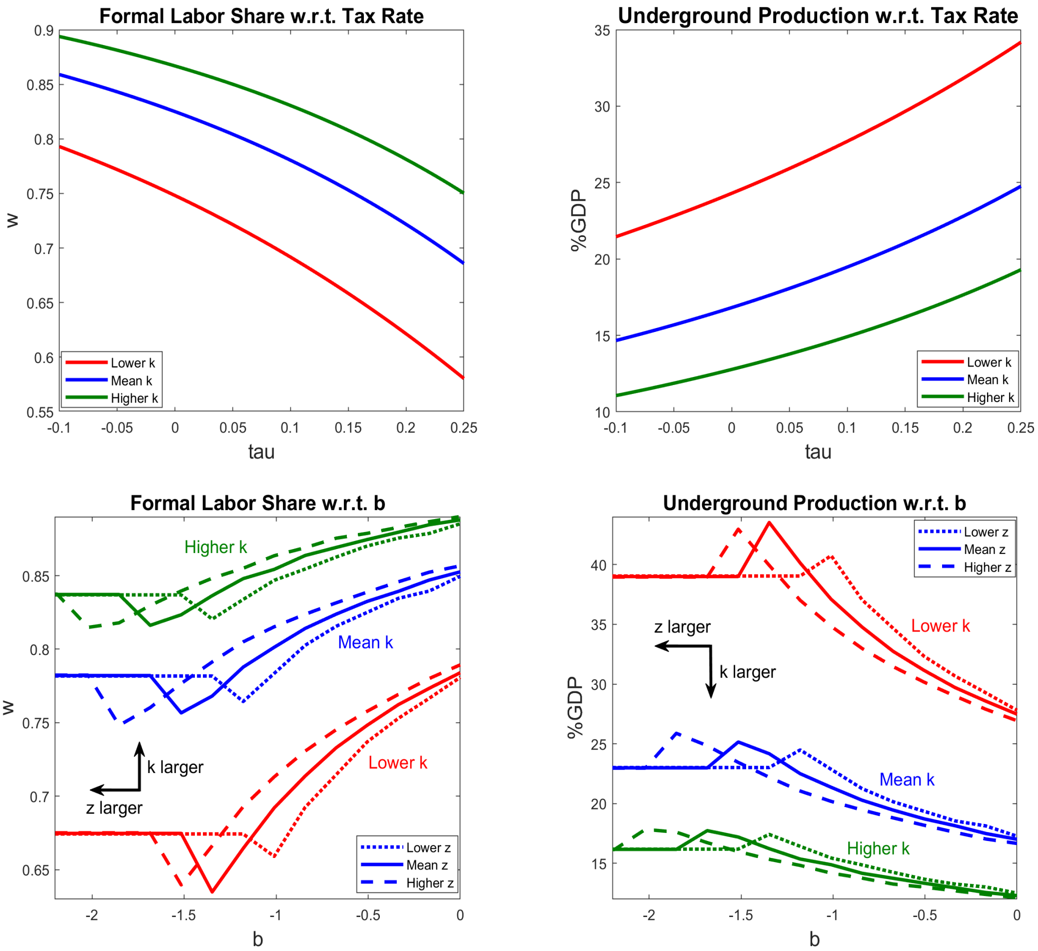
Notes: This figure displays the tax policy function $\mathcal{T}(b,k,z)$. The left panel presents $\mathcal{T}(b,k,z)$ as a function of b evaluated at the average value of capital considering different productivity realizations. Higher and lower z are $\pm 5\%$ of the mean of z. The right panel displays $\mathcal{T}(b,k,z)$ as a function of b evaluated at the average value of productivity considering different values of capital. Higher and lower k are $\pm 25\%$ of the mean of k, respectively.

is in line with the broad empirical literature on the procyclical nature of fiscal policy in emerging and developing economies, see, e.g., Talvi and Vegh (2005), Ilzetzki and Vegh (2008).

Tax hikes encourage informal activities, as the first row of Figure 5 suggests. When the tax rate increases, the share of formal labor decreases (left panel) and the size of underground economy rises (right panel). When the sovereign faces a tightening credit constraint, it has to raise taxes, but the tax hike induces informal activities. Therefore, there exists a positive relationship between sovereign debt and underground activities, which is confirmed by the second row of Figure 5. The rise in the underground economy reduces the tax base, as households in the informal sector do not pay taxes. There exists a "vicious circle" between sovereign debt and the underground economy: In bad times, the government becomes borrowing-constrained and has to raise the tax rate. Tax hikes, however, induce the private sector to engage in underground activities. In turn, a larger underground economy reduces tax revenues, which forces the government to either implement further tax hikes or to default.

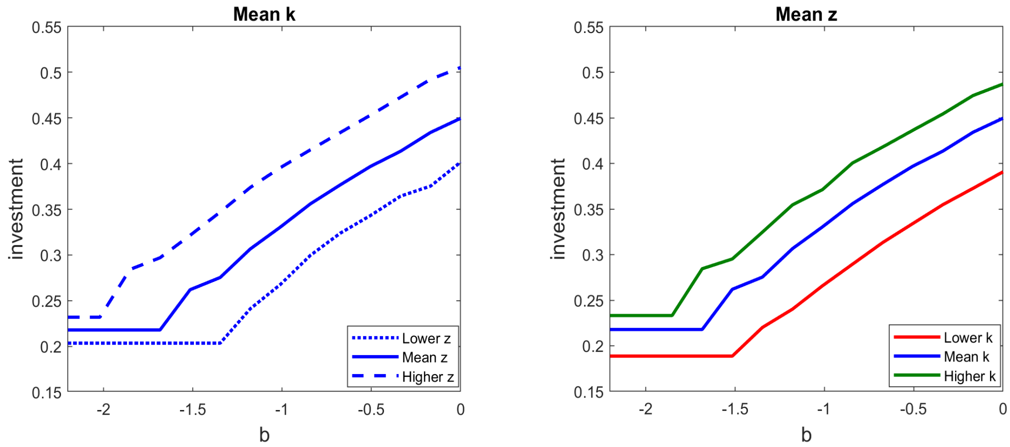
The inclusion of capital into the model further exacerbates the above-mentioned vicious circle. As Figure 6 suggests, in bad times, the private sector reduces investment in capital, which drives down the capital stock. The reduction in the capital stock has two effects. First, recalling the right panel of Figure 4, a lower capital stock leads to a higher tax rate when the sovereign is credit-constrained. Second, the size of the underground economy is larger when the capital stock is lower, as illustrated in Figure 5. Both effects amplify the vicious circle.

Figure 5: Formal Labor Share and the Underground Economy



Notes: This figure displays the formal labor share and the share of underground production. The upper panel displays the variables as functions of τ evaluated at different levels of capital. The lower panel displays the variables as functions of b evaluated at different levels of capital and productivity. Higher and lower k are $\pm 25\%$ of the mean of k, respectively. Higher and lower z are $\pm 25\%$ of the mean of z, respectively.

Figure 6: Investment



Notes: This figure displays investment as a function of b evaluated at different levels of capital and productivity. Higher and lower k are $\pm 25\%$ of the mean of k, respectively. Higher and lower z are $\pm 25\%$ of the mean of z, respectively.

4.4.3 Business Cycles and the Underground Economy

In Table 6, we report the model-generated business-cycle statistics and compare them with the empirical counterparts. The model matches the targeted moments of the Argentine data (higlighted in bold) very well. Specifically, our theoretical economy is characterized by substantial sovereign default risk, reflected by a high and volatile sovereign spread. At the same time, the size of the underground economy is non-negligible. Moreover, since the model allows for long-term debt, it generates the empirically observed debt-to-GDP ratio. In addition to the targeted statistics, our theoretical framework replicates the empirical non-targeted moments, such as the volatility of consumption relative to output. This is in line with previous studies, e.g., Arellano (2008), Cuadra et al. (2010). These studies have shown that the endogenous borrowing constraint implies that private consumption is more volatile than (official) output. Our model generates countercyclical net exports and sovereign spreads, in line with the empirical observations on typical emerging economies made by Neumeyer and Perri (2005).

Our focus is on the interaction between the underground economy and sovereign debt. As reported by the last four rows of Table 6, the underground activities are positively correlated with sovereign debt and sovereign interest spreads. Namely, the size of the underground economy and the share of informal labor are both high during times when the sovereign is facing a tight credit constraint. This finding is in line with the empirical evidence reported in Section 2. In our quantitative model, as highlighted by the policy functions in Subsections 4.4.1 and 4.4.2, increasing debt tightens the endogenous borrowing constraint and raises the sovereign's credit costs (Figure 3), which forces the government to raise the tax rate (Figure 4). However, tax hikes increase the private sector's incentives to engage in underground activities (Figure 5). This mechanism is reflected in the cyclical properties of the informal labor share and the share of underground production, which are both positively correlated with sovereign debt and interest spreads.

The cyclical properties of the informal labor share and the size of the underground economy are further driven by the procyclicality of investment. As illustrated in Figure 6, households reduce in-

Table 6: Business Cycle Statistics

Statistic		Data	Model
size of underground economy	$\mathrm{E}(y_u/y_o)$	0.23	0.23
average spread (%)	$\mathrm{E}(s)$	7.70	7.78
consumption share	$\mathrm{E}(c_o/y_o)$	0.69	0.68
government consumption share	$\mathrm{E}(g/y_o)$	0.12	0.12
investment share	$\mathrm{E}(i/y_o)$	0.17	0.17
defaultable debt to GDP	$\mathrm{E}(-b/y_o)$	0.72	0.72
default probability	$\mathrm{E}(d)$	0.9	1.7
volatility of output (%)	$\sigma(y_o)$	4.86	4.85
volatility of spread (%)	$\sigma(s)$	3.51	3.42
volatility of consumption	$\sigma(c_o)/\sigma(y_o)$	1.09	1.09
volatility of government consumption	$\sigma(g)/\sigma(y_o)$	0.41	0.88
volatility of net export	$\sigma(nx)/\sigma(y_o)$	0.30	0.90
volatility of investment	$\sigma(i)/\sigma(y_o)$	2.56	2.60
correlation of consumption	$\rho(c_o, y_o)$	0.98	0.87
correlation of government consumption	$\rho(g, y_o)$	0.67	0.87
correlation of investment	$\rho(i, y_o)$	0.98	0.78
correlation of net export	$\rho(nx, y_o)$	−0.79	−0.18
correlation of spread	$\rho(s, y_o)$	−0.80	−0.45
corr. underground size and spread	$\rho(y_u/y_o\,, s)$		0.43
corr. informal labor share and spread	$\rho(1-w\,, s)$		0.56
corr. underground size and debt	$\rho(y_u/y_o\,, -b)$		0.26
corr. informal labor share and debt	$\rho(1-w\,, -b)$		0.48

Notes: The empirical statistics ("Data") refer to quarterly Argentine data between 1994Q1 to 2001Q4. The simulation results ("Model") are averages over 1000 simulations. Each simulation has a sample size of 2000 quarters, but the first 200 quarters are abandoned. Model statistics are calculated for episodes in which no default occurs. Each non-default episode must be at least 30 quarters long. Both data and simulated series are linearly detrended. The targeted statistics are highlighted in bold.

vestment in capital when the sovereign faces a tight credit constraint. On the one hand, as discussed in Subsection 4.4.2, the reduction in investment exacerbates the vicious circle between the underground economy and the sovereign's credit constraint during bad times. On the other hand, as the informal sector does not use capital, the reduction in the capital stock reduces the labor productivity in the formal sector. The reduction in the formal labor productivity decreases the relative size of the formal sector even if w is fixed.

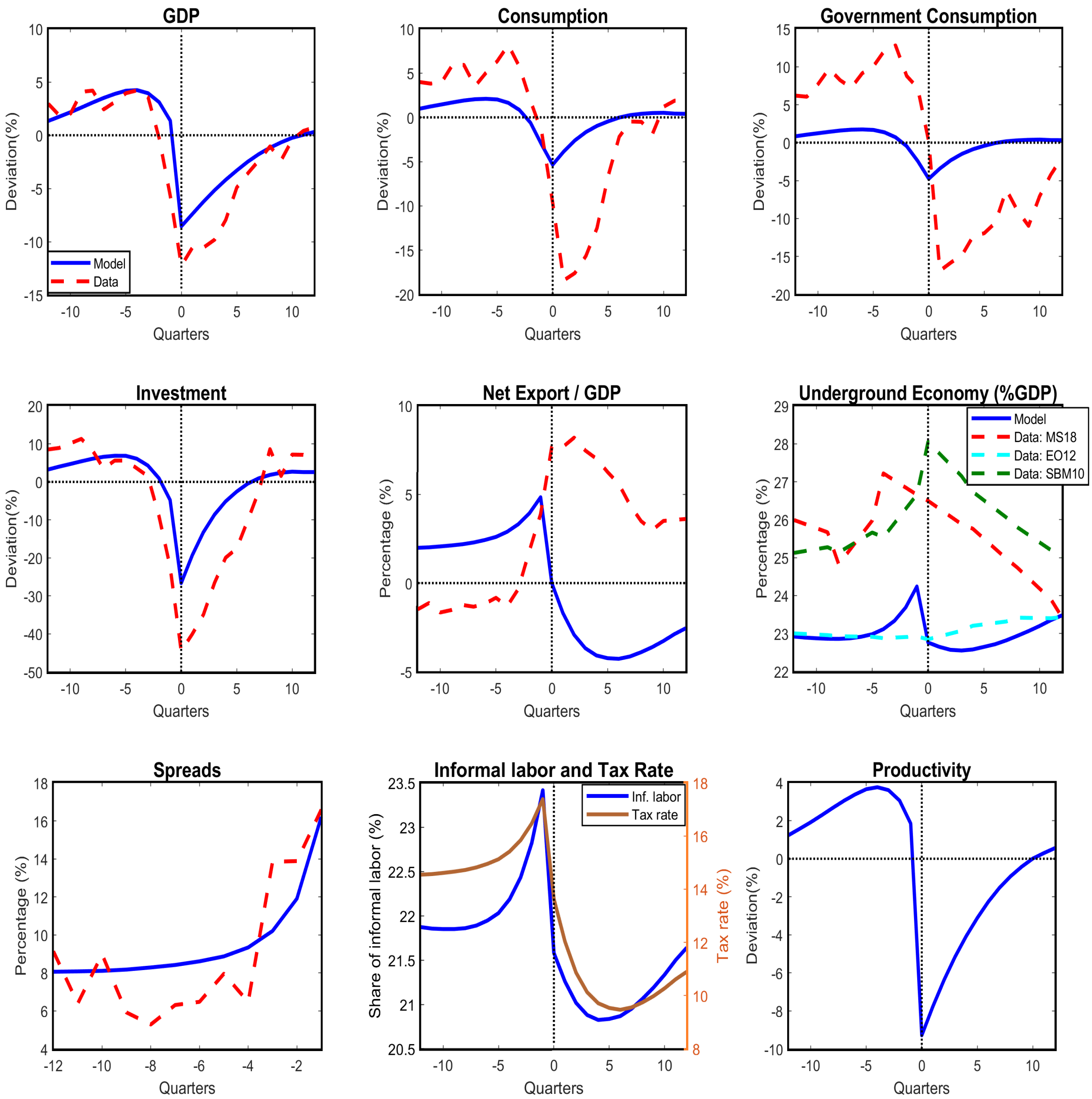
4.4.4 Sovereign Default and the Underground Economy

To understand the role of the underground economy during a debt crisis, we perform an event study and show the dynamics of the simulated economy 12 quarters before and after a sovereign default. We assume that the government has access to international financial markets in $t<0$ but defaults at date $t=0$. Both data and the simulated series are detrended using HP filter with a smoothing parameter of 1600. Each variable at each date is the average of 17,628 simulated default episodes. Figure 7 displays the results. The solid curves are model-simulated averages, while the dashed curves are Argentine data around its 2001-2002 debt crisis. The sovereign default occured in 2002Q1.

The bottom-right panel suggests that productivity $e(\cdot)$ peaks about one year prior to the default, and is followed by a sharp decline in $t = 0$. Lower productivity tightens the sovereign's borrowing constraint, which is reflected by a rising sovereign spread. The vicious circle kicks in: The tightening borrowing constraint forces the government to raise the tax rate. The tax hike induces more tax evasion and underground activities. In turn, more underground activities reduce tax revenues. Facing inadequate tax revenues, the government is forced to further raise the tax rate, which encourages even more underground production. The vicious circle keeps rolling until the period before the default. Moreover, the decreasing productivity and the tightening borrowing constraint brings down private investment in capital. As discussed earlier in Subsection 4.4.2, the falling investment during bad times further exacerbates the vicious circle. Therefore, in Figure 7, the sovereign spread, the tax rate, the share of informal labor, and the share of underground economy keep growing prior to the default. GDP and official consumption fall due to lower productivity, lower investment, higher tax rate, and the larger underground economy. Government consumption decreases because debt repayment becomes more expensive.

At some point, it becomes too costly for the government to implement further tax hikes, such that the government finds it optimal to default on the external debt obligations. After the default, the government is excluded from international financial markets, and the economy suffers from the exogeous output loss. At $t = 0$, since debt is not repaid, the government budget constraint relaxes, such that the tax rate on income can be lowered. The reduction in the tax rate encourages formal production, and hence the informal sector shrinks immediately after default. Figure 7 shows the

Figure 7: Default Events



Notes: Default events are plotted 12 quarters before and after a default takes place at $t = 0$ (except the spread). The solid lines refer to model simulations, while the dashed lines refer to the Argentine data. "Data:MS18", "Data:EO12" and "Data:SBM10" represent the estimates of the underground economy by Medina and Schneider (2018), Elgin and Öztunalı (2012), and Schneider et al. (2010), respectively. The default event in the data refers to Argentina in 2002Q1. Data and simulated series of GDP, consumption, government consumption, investment, and productivity are logged and HP filtered with a smoothing parameter of 1600.

sudden drop of the tax rate, of informal labor share, and of the underground economy at $t = 0$.

With the re-entry to international financial markets, productivity recovers. At the initial stage of the recovery, the debt burden is low and the sovereign borrows to substitute taxation. The vicious circle is now reversed and becomes a "virtuous circle": A low tax rate encourages formal activities, and a larger formal sector increases tax revenues, which allow the government to further reduce the tax rate. The drop in the tax rate amplies the positive impact on formal labor. Moreover, due to the decreasing tax rate and the rising productivity, investment recovers, which fosters the virtuous circle. As a result of the increasing productivity, the larger formal sector and the rising investment, GDP, private consumption, and government consumption recover. However, at the later stage of the recovery, the quickly built-up debt stock raises sovereign interest spreads and restricts further borrowing. Consequently, the government has to gradually raise the tax rate to an average level. The reduction in the informal activities stops, and the informal labor share and the size of the underground economy rebound to pre-default normality. However, because of the onging recovery of productivity and investment, GDP, official consumption and government spending keep increasing. The economy gradually returns to normal.

The simulated patterns of GDP, official consumption, investment, government consumption, underground econom, and the sovereign spread match their empirical counterparts during Argentina's default episode in 2001-2002. Our model fails to match the pattern of net exports. Specifically, the model predicts a trade surplus prior to the default but a trade deficit afterwards, which is contrary to the data. This is a common shortfall of sovereign debt models with long-term debt, see Gordon and Guerron-Quintana (2018). As for the dynamics of the underground economy around the default event, our model predicts a spike of the underground economy prior to the default, followed by a drop of the underground activities at the default event, as well as during the initial stage of recovery. This is in line with the estimates of the underground economy by Medina and Schneider (2018) and Schneider et al. (2010). In addition, our model predicts a gradual rise of size of the informal sector at the later stage of recovery, which is in line with the estimates of Elgin and Öztunalı (2012).

5 Conclusions

This paper studies the dynamic interaction between sovereign debt, taxation, and the underground economy. Our empirical analysis provides evidence that the size of the underground economy is positively correlated with sovereign debt and sovereign interest spreads, particularly in emerging economies.

We rationalize these empirical regularities within a quantitative model of sovereign debt and default, which incorporates an underground production sector that is hidden from the tax authorities. In an application to the Argentine economy, we show that the dynamic interaction between sovereign default risk and the underground economy creates a vicious circle: Higher sovereign interest rates make sovereign borrowing costly and force the government to raise taxes. Tax hikes, however, induce the private sector to invest less and to evade taxes by producing in the underground sector. In turn, falling tax revenues force the government to either implement further tax hikes or to default.

The vicious circle explains the time-series correlation between the underground economy and sovereign debt: Debt and interest spreads tend to be higher, when the size of the underground economy is larger. However, our empirical analysis in Section 2 also points to a cross-sectional correlation. Specifically, countries with larger underground economies tend to have higher debt-to-GDP ratios and higher sovereign interest spreads. In future research, we aim to calibrate our model to a group of countries to see whether our model provides an explanation of the cross-sectional correlations.

A Empirical Analysis

The empirical analysis is performed on two samples of countries: OECD countries and emerging economies. The countries of each sample are listed in Table 7. As of 2015, there are 34 members of the Organisation for Economic Co-operation and Development (OECD). However, Estonia has to be excluded from our sample, because of the missing data on 10-year government bond yields during 1999-2007. The sample of emerging economies consists of countries that are covered by the EMBI Global index during 1999-2007. As can be seen from Table 7, both samples lead to unbalanced panels.

Table 7: Samples

Time Window	Countries
OECD Countries (33)	
1999 – 2007 (25)	Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States
2000 – 2007 (3)	Czech Republic, Slovakia, South Korea
2001 – 2007 (2)	Mexico, Poland
2002 – 2007 (1)	Slovenia
2004 – 2007 (1)	Chile
2006 – 2007 (1)	Turkey
Emerging Economies (34)	
1999 – 2007 (20)	Argentina, Brazil, Bulgaria, Chile, China, Colombia ,Côte d'Ivoire, Ecuador, Hungary, Lebanon, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Turkey, Venezuela
1999 – 2006 (3)	Nigeria, Panama, Thailand
1999 – 2004 (2)	Croatia, South Korea
2000 – 2007 (1)	Ukraine
2001 – 2007 (3)	Egypt, Pakistan, Uruguay
2002 – 2007 (3)	Dominican Republic, El savador, Tunisia
2004 – 2007 (1)	Indonesia
2006 – 2007 (1)	Vietnam

B Data Sources, Business-Cycle Statistics, and Default Event

For the business-cycle statistics, we consider Argentine data from 1994Q1 to 2001Q4. The data start from 1994Q1, due to the availability of the EMBI. The size of the underground economy is from Elgin and Öztunalı (2012). Data on sovereign debt is from the World Bank's International Debt Statistics (IDS) database. The sovereign interest spread is measured by the JP Morgan Emerging Markets Bond Index (EMBI Global) for Argentina. All other time series are from the National Institute of Statistics and Censuses (INDEC) of Argentina.

We first seasonally adjust the data employing the Census X13 method from the U.S. Census Bureau. Afterwards, nominal variables are deflated using the GDP implied deflator, except for imports and exports which are deflated using the import and export price deflators, respectively. The series are logged and linearly detrended before calculating the business-cycle statistics in Table 6.

In the default-event study displayed in Figure 7, we use data of Argentina from 1994Q1 to 2013Q3. The data are processed in the same way as above. The annual size of the underground economy is transformed into quarterly frequency by linear interpolation. The date of default is taken as 2002Q1. In the default-event study, the data extends beyond the date of default such that the linear filter produces different magnitudes of deviations. In order to mitigate this arbitrariness, instead of the linear filter, we apply the Hodrick-Prescott filter with a smoothing parameter of 1600.

C Numerical Algorithm

The model is solved using value function iteration folling Hatchondo, Martinez and Sapriza (2010). The infinite-horizon optimization problems of the household and the government are approximately by the finite-horizon equilibrium. The value functions, the bond price function, and the policy functions are updated simultaneously in each iteration. We employ cubic-spline interpolations following Habermann and Kindermann (2007)

With the specification of the household's utility in (17), the first-order condition of the household's optimization problem (9) or (10) with respect to w writes as

$$\frac{w^{\theta}}{(1-w)^{\nu}} = \frac{(1-\theta)(1-\tau)}{(1-\nu)(1-ps\tau)} \cdot k^{\theta}. \tag{20}$$

Equation (20) suggests that w is a function of the current state k and the tax policy τ. With the functional form of the capital adjustment cost (18), the first-order condition with respect to k' writes as

$$\begin{aligned} c^{-\eta}\left[1+\kappa\left(\frac{k'}{k}-1\right)\right] = \beta \int_{z'} c'^{-\eta} \Bigg[&\theta(1-\tau')\, e(z',d')\, k'^{(\theta-1)} w'^{(1-\theta)} \\ &+ (1-\delta) + \frac{\kappa}{2}\left(\left(\frac{k''}{k'}\right)^2 - 1\right)\Bigg] \mu(z',z)\, \mathrm{d}z'. \end{aligned} \tag{21}$$

In equation (21), $\mu(z', z)$ is the transition probability of the productivity process (19). Note that the government default decision d affects the household's policy functions through the current tax policy τ, the household's expected future tax rate τ', as well as through the current and future productivity $e(\cdot)$. Given τ, τ' and k'', the household's policies for c, w an k' are obtained by simultaneously solving the nonlinear equations (20), (21) and (6).

The state of the economy is characterized by (b, k, z). To solve the model numerically, we define evenly distributed grids $b \in [\underline{b}, \overline{b}]$, $k \in [\underline{k}, \overline{k}]$ and $z \in [\underline{z}, \overline{z}]$. Let $\mathbb{V}^r_{(0)}(b, k, z)$, $\mathbb{V}^d_{(0)}(k, z)$ and $\mathbb{V}_{(0)}(b, k, z)$ be the initial guesses for the value functions. Let $\mathcal{T}^r_{(0)}(b, k, z)$ and $\mathcal{T}^d_{(0)}(k, z)$ be the initial guesses for the tax policy functions if $d = 0$ and $d = 1$, respectively. Let $\mathcal{K}^r_{(0)}(b, k, z)$ and $\mathcal{K}^d_{(0)}(k, z)$ denote the initial guesses for capital policy function. Finally, let $\mathcal{Q}_{(0)}(b, k, z)$ be the initial guesses for the bond price. The value functions, policy functions and the bond price function are updated by the following steps on each grid point:

1. Let τ^r and τ^d be the candidate tax rates and b' be the candidate for the borrowing policy function. We apply cubic-spline interpolation to obtain $\tau' = \mathcal{T}^r(b', k', z')$ and $\mathcal{T}^d(k', z')$ based on $\mathcal{T}^r_{(0)}(b, k, z)$ and $\mathcal{T}^d_{(0)}(k, z)$, and obtain $k'' = \mathcal{K}^r(b', k', z')$ and $\mathcal{K}^d(k', z')$ based on $\mathcal{K}^r_{(0)}(b, k, z)$ and $\mathcal{K}^d_{(0)}(k, z)$. We then solve the nonlinear equations (20), (21) and (6) simultaneously to obtain the household's policy functions for c, w and k'.
2. Using the household's policy functions for c, w and k obtained in the last iteration, we derive the value functions (11), (12) and (13) for each candidate value of τ^r, τ^d and b'. We apply cubic-spline interpolation for the future values $\mathbb{V}^r_{(0)}(b', k', z')$, $\mathbb{V}^d_{(0)}(k', z')$ and $\mathbb{V}_{(0)}(b', k', z')$ based on $\mathbb{V}^r_{(0)}(b, k, z)$, $\mathbb{V}^d_{(0)}(k, z)$ and $\mathbb{V}_{(0)}(b, k, z)$, and we approximate $q(b'', k'', z')$ on the right-hand side of the bond-price function (15) by a cubic-spline interpolation for $\mathcal{Q}(b', k', z')$ based on $\mathcal{Q}_{(0)}(b, k, z)$. Global search procedures are implemented to find the maximizing values of τ^r, τ^d and b'. After this step, we update the value functions $\mathbb{V}^r_{(1)}(b, k, z)$, $\mathbb{V}^d_{(1)}(k, z)$ and $\mathbb{V}_{(1)}(b, k, z)$, as well as policies $\mathcal{T}^r_{(1)}(b, k, z)$, $\mathcal{T}^d_{(1)}(k, z)$, $\mathcal{K}^r_{(1)}(b, k, z)$, $\mathcal{K}^d_{(1)}(k, z)$.
3. Using the capital policy function $\mathcal{K}^r_{(1)}(b, k, z)$ and the borrowing policy function $\mathcal{B}(b, k, z)$ otained in the last iteration, we update the bond price function $\mathcal{Q}_{(1)}(b, k, z)$. Note that we approximate $q(b'', k'', z')$ on the right-hand side of (15) by a cubic-spline interpolation for $\mathcal{Q}(b', k', z')$ based on $\mathcal{Q}_{(0)}(b, k, z)$-

We iterate until the value functions converge.

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