Relative Alpha*

Jens Carsten Jackwerth†

Anna Slavutskaya ‡

ABSTRACT

We advocate a new measure to evaluate hedge funds - relative alpha. It links each hedge fund to a group of its peers in a straightforward, semi-parametric way. We allow for omitted factors, yet do not require knowledge of the true factor structure nor do we need to estimate any factor model. We show that relative alpha outperforms traditional, absolute alpha (e.g., based on Fung and Hsieh (2001)). Relative alpha has higher explanatory power in-sample, predicts the out-of-sample performance of hedge funds, and is more persistent. Relative alpha can also be applied successfully to mutual funds.

JEL classification: G11, G12, G23.

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†Jens Jackwerth is from the University of Konstanz, PO Box 134, 78457 Konstanz, Germany, Tel.: +49-(0)7531-88-2196, jens.jackwerth@uni-konstanz.de

‡Anna Slavutskaya is from the EPFL, Swiss Finance Institute, SFI@EPFL, Quartier UNIL-Dorigny, Extranef 248, 1015 Lausanne, Switzerland, Email: anna.slavutskaya@epfl.ch

Factor models for hedge funds are ubiquitous, with the seven factor model of Fung and Hsieh (2001) being the de facto standard with more than a 1000 cites in Google Scholar by 2016.[1] Yet factor models typically only use seven to ten factors and omit all remaining factors, since they are usually estimated from hedge fund return samples of around 36 months. As a result, the explanatory power of the Fung and Hsieh (2001) return model is only 22% to 30% in terms of adjusted R^2, and we worry that there could be omitted factors. In particular, any risk premia associated with such omitted factors could show up in the factor model intercept (alpha) of the misspecified model.[2]

A related concern is, what such poorly estimated factor models do to hedge fund selection.[3] Hedge funds are typically selected by sorting the existing funds based on historical alpha. Yet sorting on the true alpha could give different results from sorting on the alpha from a misspecified model.

Finally, as alpha is often interpreted as a measure of managerial skill, we are worried as there is no predictability in traditional hedge fund alpha in our large sample of 13,597 hedge funds during February 1994 to June 2011. Alphas computed over one 36-month period have no relation with alphas computed over the subsequent 36-month period.

This paper is about improving on the above situation. The basic idea is to avoid the challenging estimation of factor model parameters (alpha and betas) based on a misspecified

[1]Alternative models are, e.g., Agarwal and Naik (2004), Jagannathan, Malakhov, and Novikov (2010), and Namvar, Phillips, Pukthuanthong, and Rau (2016).

[2]As the expected return on an asset is the sum of betas times factor premia, an omitted factor with a positive risk premium, when held long, would spuriously increase the estimated alpha of the misspecified factor model, i.e., the one without the omitted factors.

[3]See also the theoretical concerns in Levy and Roll (2016), who argue that alpha is a poor choice for constructing portfolios as it is sensitive to the (potentially misspecified) benchmark model and theoretically only holds for infinitesimal changes in allocations.

factor model, potentially suffering from omitted variables and short time series. Rather, we note that for similar hedge funds, the contribution from factor loadings (betas) times the respective factors should be similar as they share similar betas and the same factors. Then, netting one hedge fund return against the return of a close peer will leave us with the difference in alphas (hence relative alpha) and the difference in idiosyncratic components (residuals), but largely devoid of the hard to estimate inner product of betas and factors. Time-averaging the differences in returns further reduces the influence of the idiosyncratic components, which have means of zero. We thus arrive at our measure of relative alpha, which we further improve by cross-sectionally averaging over many peers while giving more weight to more similar peers. A convenient metric for similarity turns out to be the variance of return differences, which tends to be low for close peers. We take care to show this relation in a simple theoretical factor model of hedge fund returns, in a simulation study, and through 13f filings detailing the holdings of large hedge funds. Computationally, the variance of return differences is easily computed solely from hedge fund returns.

Thus, relative alpha measures the outperformance of a hedge fund over its closest peers without resorting to a particular factor model. A final hallmark of relative alpha is that it works well. We argue that relative alpha performs better than absolute alpha in all the above three areas of concern about absolute alpha: explaining hedge fund returns in-sample, predicting future performance of hedge funds, and persistence of alpha. We now detail our improvements due to relative alpha and our contribution to the literature in each of the three areas.

First, in terms of explaining hedge fund returns, we add to the literature, which has been trying to improve the omitted factor bias by adding additional factors. The factor models of Agarwal and Naik (2004), Fung and Hsieh (2001), or Namvar et al. (2016) have seven to ten factors and are thus much richer than a single market factor model or the Fama-French three factor model.[4] In particular, Fung and Hsieh (2001) is nowadays the standard factor model for hedge fund research, but even the seven factor Fung and Hsieh (2001) model only accounts in our sample for 22% - 30% (adjusted R^2) of the hedge fund return variation, depending on the sample length (12, 24, and 36 months). The Namvar et al. (2016) model performs even worse. Thus, we are concerned that the remaining more than 70% of return variation might be due to omitted factors. With a similar intuition to our paper, Jagannathan et al. (2010) add a style index to their factor model, which accounts for some commonality in the omitted factors across hedge funds in the same style.[5] While we share their desire to incorporate peer information, we argue that our approach is preferable since it avoids the estimation of the (augmented) factor model altogether, which eliminates estimation errors in the betas.

Using our relative alpha approach and interpreting the returns on the closest peers as pseudo-factors, we achieve an adjusted R^2 of 68% - 82% for the seven closest peers and 67% - 78% for the ten closest peers. These high values are not surprising since we picked the peers based on their similarity with the reference hedge fund, but they show that we achieve our goal of finding peers that are similar to our hedge funds in terms of low variances of return differences.

Second, in terms of predicting future hedge fund performance, we worry that omitted factors might bias estimated absolute alphas. Sorts based on such biased alphas might not be able to identify hedge funds, which will perform well out-of-sample. We already saw that adjusted R^2 tends to be low and Titman and Tiu (2011) document that the R^2 of the Fung and Hsieh (2001) model varies widely across hedge funds, with low R^2 funds outperforming

[4]Doherty, Savin, and Tiwari (2016) combine several existing factor models to create a pooled benchmark. Each subset of factors is weighted according to a log score criterion following the approach of Geweke and Amisano (2011).

[5]Wilkens, Yao, Jeyasreedharan, and Boehler (2015) extend this model by using the style index *orthogonal* to the standard factors as opposed to the simple style index.

high R^2 funds out-of-sample. Moreover, Bollen (2013) shows that hedge funds with low R^2 fail more often than high R^2 funds even after controlling for idiosyncratic volatility. These results would be consistent with a story, where the risky omitted factors command high premia (leading to future outperformance), while possibly carrying large downside risks (leading to fund failure).

Empirically, we find little evidence that absolute alpha predicts future hedge fund returns. The out-of-sample monthly Sharpe ratio of a portfolio constructed from the top decile of absolute alphas (estimated from rolling windows of 36 months of past returns) is only 0.32, insignificantly higher than the Sharpe ratio of 0.26 for a same size portfolio of random hedge funds (p-value of 0.11). Sorts based on the Namvar et al. (2016) model yield a monthly Sharpe ratio of 0.29. In contrast, sorts based on our relative alpha perform significantly better with a Sharpe ratio of 0.42. The advantage of relative over absolute alpha is even more drastic, when we look at the monthly Sharpe ratio of a top-minus-bottom decile strategy. Absolute alpha generates a monthly Sharpe ratio of 0.18 for the Fung and Hsieh (2001) and 0.15 for the Namvar et al. (2016) model whereas relative alpha generates a monthly Sharpe ratio of 0.61, a remarkable 3.4 to 4 times higher.

Our results are robust during non-crisis times and weaken somewhat during crisis times, as the crisis sample covers only 28 of the total 208 monthly cross sections. Results also hold up but weaken somewhat when we look at the three major hedge fund styles separately (Global macro, Equity long/short, Relative value) as the samples are much smaller now (16%, 49%, 10% of the total).

Our results are being mirrored when using alternative performance measures. As our Sharpe ratio results already suggest, mean returns are higher and volatility is lower for relative alpha compared to absolute alpha. Using the Manipulation Proof Performance Measure (MPPM) of Goetzmann, Ingersoll, Spiegel, and Welch (2007) gives significantly larger certainty equivalents for relative alpha than for absolute alpha. Measuring tail risk by

Value-at-Risk (VaR) generates significantly lower tail risk for relative alpha than for absolute alpha. Finally, the second order stochastic dominance test of Davidson and Duclos (2013) rejects that investors are indifferent between sorts based on relative versus absolute alpha and would prefer relative alpha sorts.

We find that relative alpha outperforms absolute alpha in out-of-sample sorts based on absolute alpha. This result basically remains in place when we use alternative sorting criteria. Namely, we sort according to the absolute alpha of the Agarwal and Naik (2004) factor model, the absolute alpha of the Namvar et al. (2016) factor model, the strategy distinctiveness index (SDI) of Sun, Wang, and Zheng (2012), and the manipulation-proof performance measure (MPPM) of Goetzmann et al. (2007).

We also show that our results are not qualitatively affected by changes to the methodology and the sample. For example, we vary the size of sample from which we estimate the factor models (from 36 to 24 months), increase the hedge fund holding period from one to 12 months (which corresponds to the typical lock-up period in the hedge fund industry), eliminate small hedge funds, and use de-smoothed instead of observed hedge fund returns.

We advocate the use of relative alpha for hedge funds, where the omitted factor problem is severe at about 70% of adjusted R^2 not being explained. But does relative also work for mutual funds? There, the Fama and French (1992) three factor model leaves only about a third of adjusted R^2 unexplained. Yet relative alpha still outperforms absolute alpha strongly in the top-minus-bottom decile portfolios, with more mixed results for the top and the bottom portfolios separately. Thus, relative alpha still works in a mutual fund setting, even though it was designed for hedge funds.

Third, in terms of persistence of alpha, we add to the existing literature of Ammann, Huber, and Schmid (2013) and Capocci and Hübner (2004), who show limited evidence concerning the predictability of absolute alpha. Historically measured absolute alpha has little predictive value for future absolute alpha. As alpha is often interpreted as the skill of the hedge fund manager, lack of persistence in alpha could mean that managerial skill is also not persistent.[6] Another concern is that the lack of persistence is due to the poor estimation of absolute alpha. Typical (seven to ten) factor models are often estimated with only 36 monthly returns, leading to imprecise estimates of absolute alpha.

Hoberg, Kumar, and Prabhala (2014) add an interesting perspective from the mutual fund literature, but which is still relevant to our relative alpha measure in a hedge fund setting. They define customized peer alpha as the outperformance of one mutual fund over its close competitors. The distance to competitors is based on the characteristics (e.g., size, momentum, dividend yield) of their stock holdings; all information which we do not have readily available for hedge funds.[7] The authors document persistence of their measure. While we share the basic idea of comparing funds to close peers, our measure does not need holdings information and relies solely on the readily available time series of hedge fund returns.

Our tests of alpha persistence show that relative alpha exhibits significant positive coefficients in regressions of past relative alpha on future relative alpha. For samples of 24, 48, and 72 months (the earlier half used for the past alpha estimation, the latter half used for the future alpha estimation), our regressions have adjusted R^2 of 24%, 29%, and 27%. Results for absolute alpha are sobering with being either insignificant or having even marginally significant *negative* coefficients for the Fung and Hsieh (2001) and the Namvar et al. (2016) models with virtually no explanatory power (adjusted R^2 less than 0.004).

[6]For mutual funds, Berk and Green (2004) argue that competition should eliminate such skill related alpha. In Berk and van Binsbergen (2013) they thus argue that mutual funds should not exhibit persistence in alpha. However, Glode and Green (2011) show theoretically for hedge funds that potential information spillovers (associated with innovative trading strategy or emerging sector) could lead to persistent alpha after all.

[7]Hedge fund holdings are unknown but for the 13f reports, which only apply to US equity holdings of large funds (> \$100 million).

Furthermore, we design a simulation study to analyze under which circumstances relative alpha works best. Relative alpha works well even in the presence of omitted variables, while absolute alpha performs poorly in that situation. A large cross-section of hedge funds and longer time series of returns contribute to the superior performance of relative alpha. Yet the strong performance of relative alpha remains even when we drop half the cross section of peer funds or shorten the time series from 36 to 24 months.

We next develop our new relative alpha measure in Section I. All data are presented in Section II. Results and robustness checks follow in Section III. We study our relative alpha measure via a simulation in Section IV and conclude with Section V.

I. Relative Alpha

We assume that there exists a (possibly very large) factor model with uncorrelated error terms, which explains hedge fund returns.[8] The factors can be correlated with each other, but we do not allow factors to be linear combinations of other factors. We do not limit ourselves to the seven or eight factors of Fung and Hsieh (2001) or Agarwal and Naik (2004) but include otherwise omitted factors beyond those standard factors. Thus, our assumed factor model would have high explanatory power but for the error term, i.e., an R^2 of close to one. Obviously, we cannot name these omitted factors, but we do not need to; we argue below that we can still assess our performance measure relative alpha without the explicit knowledge of the full factor model by essentially netting out much of the unknown factor structure. The complete factor models for hedge funds i and j are then as follows:

[8]Empirically, a principle component based attribution model, where we are using sufficiently many principal component so that we achieve a high R^2, would fit the bill.

$$r_{it} - r_{f,t} = \alpha_i + \sum_{l=1}^{L} \beta_{i,l} F_{l,t} + \varepsilon_{i,t}, \tag{1}$$

$$r_{jt} - r_{f,t} = \alpha_j + \sum_{l=1}^{L} \beta_{j,l} F_{l,t} + \varepsilon_{j,t}, \tag{2}$$

where $(r_{i,t} - r_{f,t})$ is the excess hedge fund return of hedge fund i at time t, $F_{l,t}$ is the factor l at time t, $\beta_{i,l}$ is the risk exposure of hedge fund i to factor l, and $\varepsilon_{i,t}$ is a mean zero error term for hedge fund i. Definitions for hedge fund j are similar.

To neutralize the impact of the factor exposure, we take expectations of the differences in returns. If hedge funds i and j implement identical strategies (i.e., they load on the same risk factors $F_{l,t}$ and have $\beta_{i,l} = \beta_{j,l}$), then their factor loadings cancel, leaving only differences in alphas as the error terms have mean zero:

$$E[r_{i,t} - r_{j,t}] = \alpha_i - \alpha_j + \sum_{l=1}^{L} (\beta_{i,l} - \beta_{j,l}) E[F_{l,t}] + E[\varepsilon_{i,t} - \varepsilon_{j,t}] = \alpha_i - \alpha_j. \tag{3}$$

We will use this difference in hedge fund i's alpha from hedge fund j's alpha to measure the outperformance of one hedge fund over its peer.

Now clearly, hedge funds typically do not have perfectly identical betas and $\beta_i - \beta_j$ may have non-zero entries. Yet we argue that Equation (3) still hold approximately as long as the entries of $\beta_i - \beta_j$ are small in absolute value. Note, that the mean of each element of $\beta_i - \beta_j$ is zero across all pairings i, j as each hedge fund will appear just as often on the left and on the right of the minus sign. Thus, the average value of the approximation errors $\sum_{l=1}^{L} (\beta_{il} - \beta_{jl}) E[F_{l,t}]$ across all pairing i, j is also zero. In order to minimize the influence of the approximation errors, we compute the relative alpha of a reference hedge fund j (Δ_j) as the weighted average of the differences in returns of j against all other hedge funds i:

$$\Delta_j = \sum_{i \neq j} w_i E[r_{i,t} - r_{j,t}] \approx \alpha_j - \sum_{i \neq j} w_i \alpha_i, \tag{4}$$

with $\sum w_i = 1$.

Such weighted average reduces the total approximation error as the individual approximation errors ($\sum_{l=1}^{L} (\beta_{i,l} - \beta_{j,l}) E[F_{l,t}]$) partially cancel.[9] The remaining question is, how to weight the different hedge funds. We noted above that the approximation error has mean zero across all pairings of i and j. Yet the variability of the approximation error also matters as more similar peers of hedge fund j would lead to less variable approximation errors and thus to less variability in our relative alpha measure in Equation (4). Formalizing this idea, we would ideally base our weights on the (element-wise) squared approximation error, i.e., the weighted sum of squared differences in betas times the squared expected factors:

$$\sum_{l=1}^{L} (\beta_{i,l} - \beta_{j,l})^2 E[F_{l,t}]^2. \tag{5}$$

Yet, as we do not observe the betas nor the many (potentially unknown) factors of the model, we suggest to instead use the variance of return differences, which has a related structure and can be simply computed from just the returns without any knowledge of the betas nor the factors of the model:

$$Var[r_{i,t} - r_{j,t}] = (\beta_i - \beta_j)' Cov[F_{l,t}] (\beta_i - \beta_j) + \sigma_i^2 + \sigma_j^2, \tag{6}$$

[9] Unless all the mean zero approximation errors happen to have same sign whence they simply add.

where β_i (β_j) is the $(L \times 1)$ vector of risk exposures of hedge fund i (j), $Cov[F_{l,t}]$ is the $(L \times L)$ variance-covariance matrix of the factors (assumed to be bounded from above), and σ_i^2 (σ_j^2) is the variances of the error term $\varepsilon_{i,t}$ ($\varepsilon_{j,t}$).

We now argue in three different ways that a low variance of return differences in Equation (6) relates to a small squared approximation error in Equation (5). First, we introduce a simple model, second a simulation, and third we present empirical evidence based on the 13f holdings data for large hedge funds. Plus, our results in Section III show that our trick works really well empirically.

A simple model of hedge funds

We assume that hedge funds load on the factors based on some average beta vector plus idiosyncratic variations in those betas, where all variations are iid normally distributed. Further, we assume that the L factors are known with means $E[F]$ and covariance matrix $Cov[F_{l,t}]$. Finally, we assume the residuals to be i.i.d. normally distributed with mean zero and some (possibly different) variances. If hedge funds follow such simple factor model, then the mean approximation error is zero.

PROPOSITION 1: *Let us assume that* $\beta_{i,l} = \beta_l + \eta_{i,l}$ *with* $\eta_{i,l} \sim \mathcal{N}(0, \sigma_\eta^2), \forall i$. *Further assume that* $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2), \forall i$. *We show that*

$$E\left[\sum_{l=1}^{L} (\beta_{i,l} - \beta_{j,l}) E[F_{l,t}]\right] = \sum_{l=1}^{L} E[\eta_{i,l} - \eta_{j,l}] E[F_{l,t}] = \sum_{l=1}^{L} 0 \times E[F_{l,t}] = 0.$$

Further, the squared approximation error positively covaries with the variance of return differences. Thus, we can find close peers by searching for hedge funds with low variances of differences in returns. We show this result in the following proposition with the proof being relegated to Appendix.A.

PROPOSITION 2: *Using the above assumptions, we show that*

$$Cov\left[Var[r_{i,t}-r_{j,t}],\sum_{l=1}^{L}(\beta_{i,l}-\beta_{j,l})^2E[F_{l,t}]^2\right]=(3-1)(2\sigma_\eta^2)^2\sum_{l=1}^{L}E[F_l]^2Var[F_l]>0.$$

Simulation of hedge fund returns

In Section IV (see for details), we use our main hedge fund database and assume that the first 30 principal components are the true factor model for all hedge funds. Regressions based on the true model yield an adjusted R^2 of 95% and give us the true alpha, betas, and residuals. We then simulate 100 different time series of factors (keeping the cross-sections of the factors at a point in time constant). Using the alphas, betas, and random draws from the residuals, we can create 100 simulated panels of time-series for each hedge fund out of a large cross section of hedge funds. As in our theoretical model, we show that the expectation of the approximation error $\sum_{l=1}^{L}(\beta_{i,l}-\beta_{j,l})E[F_{l,t}]$ is -0.0005 and insignificantly different from zero with a p-value of 0.65. We also run the regression of the variance of return differences $Var[r_{it}-r_{jt}]$ on the squared approximation error $\sum_{l=1}^{L}(\beta_{i,l}-\beta_{j,l})^2E[F_{l,t}]^2$. The slope coefficient turns out to be positive at 0.7819 (significant with a p-value of 0.03).

13f filings

For some 710 large hedge funds with more than \$100 million in assets under management, we have information on their quarterly long holdings of US equities. We now interpret each of these securities as a single factor and use the value-weighted allocations to those securities as factor loadings (betas). Thus, we do not need to estimate the factor model but observe it directly from the holdings information. We show in Section III that the mean approximation error is insignificantly different from zero (0.0010 with a p-value of 0.89) and that the variance of return differences is positively correlated with the squared approximation error (slope of 0.8366 with a p-value of 0.07). Note that results are biased against us since some hedge funds have holdings, which do not need to be reported on form 13f. As a result, finding a significant slope is harder in the presence of omitted holdings.

Summing up, we conclude from our model, from our simulation, and our study of 13f holdings that we can use the variance of return differences as a measure of similarity between hedge funds. We next specify our weights in Equation (4), which we base on our similarity measure. We suggest a Gaussian kernel weighting:

$$w_i = \frac{K\Big(Var[r_{i,t} - r_{j,t}]/h\Big)}{\sum_{i \neq j} K\Big(Var[r_{i,t} - r_{j,t}]/h\Big)}, \tag{7}$$

where $K(\cdot)$ is a Gaussian Kernel and h is the bandwidth according to the Silverman (1986) rule of thumb. A low variance of return differences (i.e., a higher similarity) leads to a high weight for the peer fund, with h controlling the relative importance of each peer fund. Yet, our results do not change much if we use bandwidths from a fifth to five-times of the value of the Silverman (1986) rule of thumb. We are now ready to compute relative alpha as:

$$\Delta_j = \sum_{i \neq j} w_i E[r_{i,t} - r_{j,t}] = \frac{\sum_{i \neq j} K\Big(Var[r_{i,t} - r_{j,t}]/h\Big)\, E[r_{i,t} - r_{j,t}]}{\sum_{i \neq j} K\Big(Var[r_{i,t} - r_{j,t}]/h\Big)}. \tag{8}$$

Kernel estimates are biased on the boundary of the data and we suffer from this problem as we evaluate the kernel estimate at a point where the variance is zero based on variances which are all positive. Thus, we show in our robustness section that results do not change when we use the local regression technique proposed by Hastie and Loader (1993), which better accommodates the boundary bias.

II. Data

For our hedge fund data, we rely on the MOAD database described in Hodder, Jackworth, and Kolokolova (2014). MOAD is a merged database of six commercially available databases (CISDM, Barclays, TASS, HFR, Altvest, and Eurekahedge). We use only USD-denominated, net-of-fees returns with at least 36 months of historical returns which leaves us with 13,597 hedge funds. Our sample runs from February 1994 until June 2011. The descriptive statistics of our sample are presented in Table I, Panel A. We document excess kurtosis and left-skewness in hedge fund returns, suggesting that returns are often not normally distributed. The monthly Sharpe ratio is 0.18.

[Table I about here]

Hedge funds differ from other asset classes in many respects. One of them is the absence of strict regulation. This leads to database biases as reporting is voluntary. We address those biases as follows. First, our joint database is free of survivorship bias because it contains both live and dead funds. Second, to control for the instant history bias, we delete the first 12 months of each hedge fund's returns. We compute our main results based on the reported returns as we find them in the database (although we run a robustness check with de-smoothed returns).

We obtain mutual fund data from the Morningstar database. We eliminate mutual funds with less than 36 months of historical returns. The sample runs from February 1994 until June 2011. The descriptive statistics are summarized in Table I, Panel B. Mutual fund returns are characterized by excess kurtosis and left-skewness but to a lesser extent than hedge funds. The monthly Sharpe ratio is 0.09. Overall, mutual funds demonstrate smaller first and second moments compared to hedge funds.

For additional tests, we use section 13f quarterly filings to the SEC, on which large hedge funds (with assets under management worth more than $100 million) report their long positions in US equity markets, ADRs, put and call options, and convertible notes. These positions, however, do not include short sales, cash, or any other assets. The detailed description of data shortcomings can be found in Agarwal, Jiang, Tang, and Yang (2013).[10] Our sample covers 710 hedge funds from October 2001 until April 2011.

We further use the seven factors of the Fung and Hsieh (2001) model which are available at David A. Hsieh's Hedge Fund Data Library.[11] Option-based factors from Agarwal and Naik (2004) were graciously provided by the authors. We thank Kuntara Pukthuanthong for sharing data on the Namvar et al. (2016) factors. For the mutual funds, we use the three factors of the Fama and French (1992) model, which we downloaded from Kenneth R. French's website.[12]

III. Methodology and results

We now present our results where we first investigate the explanatory power of relative and absolute alpha with respect to in-sample hedge fund returns. Next, we analyze their ability to predict future performance. We continue with an investigation of the persistence of relative alpha versus absolute alpha.

A. *Explaining hedge fund returns in-sample*

We are concerned about the low explanatory power of traditional factor models, such as Fung and Hsieh (2001), as omitted factors could bias the estimated alpha. With average adjusted R^2 of 22%, 28%, and 30% at the 12, 24, and 36-months horizons, the seven factor

[10] We are very thankful to Achim Mattes and Olga Kolokolova for providing us with the data.

[11] https://faculty.fuqua.duke.edu/∼dah7/HFData.htm

[12] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Fung and Hsieh (2001) model leaves much variation in returns unexplained (see Table II, column FH).

[Table II about here]

For our relative alpha measure, we cannot directly obtain R^2. Instead, we use the returns of the seven closest peer funds as pseudo factors. Then, we are able to repeat the above exercise of computing average adjusted R^2, which are significantly higher at 82%, 67%, and 68%, respectively. We do not want to oversell our results, since we picked the closest peers exactly for similarity. Still, we picked the peers well as they explain more than two-thirds of the return variation. That bodes well for our relative alpha calculations, where the quality of the results will depend on how effective the peers cancel the products of factor loadings and factors of the reference hedge fund.

We repeat the estimation using the ten factor Namvar et al. (2016) model and the ten closest peers. The Namvar et al. (2016) model (22%, 24%, and 24%) performs even worse than the Fung and Hsieh (2001) model, while using the ten closest peers (78%, 68%, and 67%) performs similarly to using the seven closest peers as pseudo factors.

B. Predicting future performance

We next compare the out-of-sample performance of sorted portfolios based on relative and absolute alphas. We use 36-month rolling windows to estimate relative and absolute alpha. The correlation between relative alpha and absolute Fung and Hsieh (2001) alpha is on average 0.62 and ranges between 0.24 and 0.81, depending on the rolling window length of 12, 24, and 36 months. Based on relative and absolute alphas separately, we sort the hedge funds into top and bottom deciles and form equally weighted portfolios. We record returns of top, bottom, and top-minus-bottom portfolios in the 37^{th} month and repeat the procedure while moving one month forward. We take care of look-ahead bias by recording a zero return instead of a missing return in case a hedge fund is delisted.[13]

Predicting future performance in the base case

[Table III about here]

The future performance of top portfolios sorted on relative alpha is superior to sorts on absolute alpha, see Table III, columns Top Portfolio REL and FH. Both sorts deliver similar mean returns of about 1% per month while the volatility is significantly lower for relative alpha (2.08%) in comparison to Fung and Hsieh (2001) (2.47%, p-value of 0.01), which is reflected in a significantly higher Sharpe ratio (0.42 for the top relative alpha portfolio versus 0.32 for Fung and Hsieh (2001)). The difference in Sharpe ratios is significant (p-vaue of 0.01), where we apply the heteroskedasticity and autocorrelation robust inference of Andrews (1991).

We confirm these results for top portfolios when using performance measures other than the Sharpe ratio. The Manipulation Proof Performance Measure (MPPM) of Goetzmann et al. (2007) for relative alpha (9.59% annualized) is significantly higher than the MPPM for Fung and Hsieh (2001) (8.39%, p-value of 0.00).[14] The Value-at-Risk (VaR; with a 5%

[13]Among the top portfolios, around 0.7% of all hedge funds are missing the out-of-sample return. Among bottom portfolios, the number is around 3%. There is little variation in these percentages across relative versus absolute alpha. During crisis periods, the percentage rises to almost 4%.

[14]We test the difference in MPPM by using a bootstrap procedure. We draw 1000 bootstrap samples with replacement from the time series of the original portfolios and compute the MPPM from these samples. We use the bootstrapped standard errors as the standard deviation of the MPPM test statistic.

tail probability) is significantly lower for relative alpha (-1.40%) than for absolute alpha (-2.17%, p-value of 0.01).[15] We finally test if all risk averse investor prefer the top relative alpha portfolio to the top absolute alpha portfolios. We do so by testing if the top relative alpha portfolio stochastically non-dominates (in the second order) the top absolute alpha portfolio. The SSD test of Davidson and Duclos (2013) marginally cannot reject the null with a p-value of 0.12.[16]

We find for the top portfolios that sorts based on relative alpha better predict future performance than sorts based on absolute alpha. For the bottom portfolios, sorts based on relative alpha are still significantly better than portfolios based on absolute alpha sorts when we look at volatility, MPPM, and VaR. The differences are insignificant for the Sharpe ratio and SSD. Yet when turning to the top-minus-bottom portfolios, relative alpha performs much better than absolute alpha. The Sharpe ratio is now 3.4 to 4 times higher than for absolute alpha (p-value of 0.00). Volatility, MPPM, VaR, and SSD are all highly significant (p-values of 0.00, but for MPPM with 0.04).

Predicting future performance using 13f holdings

So far, we assume that hedge fund returns can be explained by a (possibly large) factor model. Typically, we do not know those factors, but hedge fund holdings could serve as a set of factors. We could treat each security as a factor and use its value-based weight as its beta. The return on the portfolio of holdings should be close to the hedge fund return. Differences can arise due to fees and if the holdings are incomplete or time-varying.

Indeed, large hedge funds with more than $100 million in assets under management need to report their long holding of (mainly) U.S. equity in so-called 13f filings with the SEC. We

[15] We test the difference in value at risk by the method proposed in Wilcox and Erceg-Hurn (2012).

[16] We apply the Davidson and Duclos (2013) test based on the t-statistic. We simulate the distribution of the test statistics in order to account for small samples.

collected a quarterly record of such 13f filings from October 2001 until April 2011 for 710 hedge funds. Then, we investigate the approximation error in line with our propositions. First, we depict the distribution of approximation errors $\sum_{l=1}^{L}(\beta_{il}-\beta_{jl})E[F_{l,t}]$ in Figure 1. As expected, it is centered close to zero at 0.0010 (p-value of 0.89). Second, we run a linear regression of the variances of the return differences against squared approximation errors. We find a significantly positive slope of 0.8366 (p-value of 0.07). In line with proposition 2, we interpret this as evidence that we can find close peers by searching for hedge funds with low variances of differences in returns. Our results stay robust with respect to winsorizing. We conclude that our propositions are in line with the empirical data for select hedge funds, for which we can construct a factor model based on observed holdings.

[Figure 1 about here.]

Predicting future performance using different methodologies

We start assessing the robustness of our results by changing the methodology of the base case, where we use all hedge funds and not only the large hedge funds with 13f filings. For one, we shorten the size of the rolling window from 36 to 24 months. We expect our estimates to be less precise due to the reduction in sample size. Indeed, the results in Table IA.1 show that relative alpha still performs better than absolute alpha, but we lose three significant results at the 5% level of the twelve significant results for the base case in Table III.

Next, we correct for the boundary bias of the kernel estimates by using locally weighted regressions as proposed in Hastie and Loader (1993). Here, the twelve significant results in Table IA.2 are very close to the base case.

Since hedge funds may have lock-up periods, investors might not be able to withdraw their investment every month and reallocate to a new hedge fund. We repeat our main results while increasing the portfolio holding period from one to twelve months. Our results in Table IA.3 hardly change compared to the base case.

Getmansky, Lo, and Makarov (2004) find that return smoothing can affect hedge fund returns. We correct for potential MA(2) components and use the de-smoothed returns instead. The results in Table IA.4 weaken for the top portfolios but stay as strong as ever for the top-minus-bottom portfolios. Still, we come in with ten significant results compared to twelve in the base case. Most likely, the estimation of the MA(2) model introduces additional estimation error to both the reference hedge fund and all potential peers. Thus, relative alpha is now saddled with estimation error, which we carefully avoid so far.

Predicting future performance using different samples

Our results are also robust to a number of changes to the sample. We start by considering a sample consisting of the NBER crises periods from March 2001 to November 2001 and from December 2007 to June 2009. For the crises periods, the sample shrinks drastically from 208 monthly cross sections to just 28. We thus expect our results to suffer and this indeed happens. During the crises periods, we find four significant results in Table IA.5, Panel A, down from twelve for the base case. Still, for the top-minus-bottom portfolios, relative alpha performs significantly better than absolute alpha for volatility, MPPM, and SSD (p-values of 0.05, 0.00, and 0.07). The non-crisis results in Table IA.5, Panel B hardly change from the base case.

After changing the time-dimension of the sample, we next change the size of the cross section of hedge funds. First, we eliminate hedge funds, which are closed to new investment, as investors might not be able to allocate money to these hedge funds. The results in Table IA.6 with ten significant results are almost as strong as the base case with twelve significant results. As it might also be hard for investors to allocate funds to very small hedge funds, we eliminate hedge funds with assets under management of less than $20 million as proposed in Kosowski, Naik, and Teo (2007). Again, with twelve significant results in Table IA.7, the situation does not change much compared with the base case.

Predicting future performance using other sorting measures

To strengthen our argument, we repeat the analysis by sorting hedge funds into decile portfolios based on alternative performance measures. All results are based on 36-month rolling windows and on recording the subsequent out-of-sample returns. As performance measurements, we use the the absolute alphas of Agarwal and Naik (2004) (AN) and Namvar et al. (2016) (NPPR), the strategy distinctiveness Index (SDI) of Sun et al. (2012), and the manipulation-proof performance measure (MPPM) of Goetzmann et al. (2007). The relevant formulas and factors are collected in Appendix.B and Appendix.C.

All alternative performance measures perform worse than relative alpha, see Table IV, with Panel A reporting the top portfolio results, Panel B the bottom portfolio, and Panel C the top-minus-bottom portfolio. In our base case (column REL in each of the three panels), we had twelve significant results of relative alpha against the absolute alpha of Fung and Hsieh (2001). The corresponding numbers of significant results are eight for AN, eleven for NPPR, twelve for SDI, and eleven for MPPM. So we perform just about as strong against the alternative performance measures as against Fung and Hsieh (2001). A closer look at the results for Agarwal and Naik (2004) shows that the weakening stems from fewer significant results for the top portfolio, while the bottom portfolios and the top-minus-bottom portfolios perform as strongly as ever.

[Table IV about here]

Predicting future performance for different investment styles

We also repeat the study within investment styles. Table V shows the results for top, bottom, and top-minus-bottom portfolios for the three investment strategies with more than 10% share of the total cross section: global macro (16% share in Panel A), equity long/short (49% in Panel B), and relative value (10% in Panel C). Results for global macro stay with twelve significant results just as strong as as they are in the base case. Somewhat surprisingly do our results weaken somewhat for the large group of equity long/short hedge funds, which exhibit eight significant results. The losses of significance are concentrated in the top portfolios with bottom and top-minus-bottom portfolios almost keeping their significance counts. For relative value, we have weaker results with seven significant results. Yet, the sample dropped drastically in size to only 10% of the cross section, thus making it harder to econometrically establish significance.

[Table V about here]

Predicting future performance for mutual funds

Even though we designed relative alpha to be used for hedge funds, we were curious how well it fares when used for mutual funds. For mutual funds, the Fama and French (1992) model explains about two-thirds of the return variation, whereas the Fung and Hsieh (2001) model explains only about one-third of the return variation for hedge funds. Thus, as relative alpha is specifically designed to deal with omitted factors, its advantage for mutual funds is less obvious. We repeat our study on mutual funds and present our results in Table VI. We still find seven significant results, with the top-minus-bottom portfolios performing strongly, while the results for top and bottom portfolios weaken somewhat. Our results suggest that the benefit of relative alpha is less pronounced for mutual funds than for hedge funds, but still present in the data. Repeating our study for the crises and non-crises periods separately does not change our results from the base case for mutual funds, see Table IA.8.

[Table VI about here]

C. *Persistence of alpha*

To analyze the persistence of alpha, we adopt a methodology commonly used in the hedge fund literature. We consider consecutive 72-month periods[17], starting with the 1^{st}, 37^{th}, 73^{rd}, ... observations. Each of these periods is divided into two 36-month sub-periods: a formation period (1-36^{th} months) and an evaluation period (37-72^{nd} months). For each hedge fund i which survives the whole 72-month period, we compute relative alpha during the formation ($\triangle_{1i}$) and the evaluation ($\triangle_{2i}$) periods and estimate the following regression:

$$\triangle_{2i} = a_\Delta + b_\Delta \triangle_{1i} + \omega_i, \tag{9}$$

where a_Δ, b_Δ are the parameters to be estimated, and ω_i is an error term. We stack all observations for the different non-overlapping periods and then run the joint regression. Persistence in relative alpha is determined by a significantly positive coefficient b_Δ.

The persistence study is repeated for absolute alphas from the seven factor Fung and Hsieh (2001) model and the ten factor Namvar et al. (2016) model:

$$\alpha_{2i} = a_\alpha + b_\alpha \alpha_{1i} + v_i, \tag{10}$$

where $\alpha_{1i}(\alpha_{2i})$ are the alpha estimates in the formation (evaluation) period for fund i; a_α, b_α

[17]While 72 months (36+36) is a typical sample used in the hedge fund literature, it creates a significant look-ahead bias as only funds are included, which survived for all 72 months. Such filtering also eliminates many hedge funds. Consequently, we also use 48 (=24+24) and 24 (=12+12) months samples. We have in our database some 7,145 hedge funds with 72 months of observations, 11,002 with 48, and 16,011 with 24 months.

are the parameters to be estimated, and v_i is an error term.

We know from Ammann et al. (2013) and Capocci and Hübner (2004) that there is little evidence on hedge fund absolute alpha persistence. We are therefore interested if relative alpha is more persistent than absolute alpha. We present the results for the 72-month period in Table VII, Panel C. Relative alpha is strongly persistent with a t-stat of 24.03 while absolute alpha is actually anti-persistent with negative coefficients for both Fung and Hsieh (2001) and Namvar et al. (2016) and t-stats of -2.45 and -2.65. Shortening the period to 48 months (24 months each in the formation and the evaluation period) in Panel B again shows strong persistence for relative alpha with a t-stat of 16.46, while Fung and Hsieh (2001) stays negative (t-stat of -2.93) and Namvar et al. (2016) turns insignificant. Further shortening the period to 24 months still shows strong persistence for relative alpha (t-stat of 13.43), while Fung and Hsieh (2001) turns insignificant and Namvar et al. (2016) shows anti-persistence (t-stat of -2.50).

[Table VII about here]

IV. Why does relative alpha work?

In order to see why relative alpha performs so well, we turn to a simulation study. In our simulation, we know the true alphas and we can thus confirm that our two propositions hold for our simulation. Further, we can vary the size of the cross section and the length of the time series to assess how relative and absolute alpha are affected.

We simulate hedge fund returns based on the principal components extracted from the observed hedge fund returns. For a time series of 36 months and a cross-section of 2,349 hedge funds, we use the first 30 principle components, which explain 95% of the total return variation, as our true 30-factor model. We estimate alpha, betas, and residuals for each hedge fund based on the 30-factor model. We assume that those estimated parameters are the true parameters for our simulation. The true alphas are distributed around zero with mean 0.0019 and standard deviation 0.0324. We simulate 100 different scenarios. Here, we resample with replacement the time series of factor realizations, while keeping the cross-section of the 30 factors intact. Based on the true alpha and betas and random draws with replacement from the residuals, we can generate new time series for all hedge funds during each scenario.

Our simulation seamlessly incorporates omitted factors. Estimating the seven factor Fung and Hsieh (2001) model for each hedge fund, the typical adjusted R^2 is only 30-40%, i.e., much information of the true 30 factor model (with an adjusted R^2 of 95%) is omitted.

We first simulate our full sample with 2,349 hedge funds and 36 monthly returns each. Based on the relative alpha formula in Equation (8), we can substitute the true alpha differences instead of the differences in returns, $E[r_{i,t} - r_{j,t}]$. We call this the true relative alpha:

$$\Delta_j^{true} = \sum_{i \neq j} w_i(\alpha_{i,t} - \alpha_{j,t}) = \frac{\sum_{i \neq j} K\left(Var[r_{i,t} - r_{j,t}]/h\right)(\alpha_{i,t} - \alpha_{j,t})}{\sum_{i \neq j} K\left(Var[r_{i,t} - r_{j,t}]/h\right)}. \tag{11}$$

We average the true relative alphas and record the value of -0.0005 in Table VIII, Panel A, Full Sample. The average estimated relative alpha is 0.0009, insignificantly different from the true value with a p-value of 0.21. For absolute alpha, the average true value is 0.0015, while the average estimated alpha based on the Fung and Hsieh (2001) model is 0.0051, significantly different with a p-value of 0.00.

[Table VIII about here]

We next check on our two propositions. First, we suggested that the approximation error

$\sum_{l=1}^{L}(\beta_{i,l}-\beta_{j,l})E[F_{l,t}]$ has a theoretical mean of zero and thus allows us to use expected differences in returns to approximate differences in alpha. In Figure 2, we depict the distribution of simulated approximation errors. It is a rather symmetric distribution and centered at -0.0005, insignificantly different from zero with a p-value of 0.65.

[Figure 2 about here.]

Second, we argued that the squared approximation error increases in the variance of return differences. Indeed, the slope of the variance of the return differences against squared approximation error is significantly positive at 0.7819 (with a p-value of 0.03). Thus, we are justified to give more weight to close peers (with low variances of return differences) in order to reduce the squared approximation error and, thus, to estimate relative alpha more precisely.

We now have our simulation in place, with a typical sample size, a realistic omitted factor structure, and our two propositions hold. Next we turn to answer, why relative alpha works so well. For one, the computation of relative alpha in Equation 8 needs a reasonable number of close peer funds. We will thus vary the number of hedge funds in the cross section. Second, closeness is measured as the variance of return differences and our estimate thereof will be more precise the longer the time series of returns is. We thus vary the number of observations available in the time series.

Varying the number of hedge funds

We first compared relative and absolute alpha based on the simulated full sample with N=2,349 hedge funds. The error between the true and the estimated average values are -0.0014 for relative alpha and -0.0036 for absolute alpha, see Table VIII, Panel A, Full Sample. We next reduce the number of hedge funds, which can be used as peers for relative alpha. That should make it harder for relative alpha and absolute alpha should not be affected much. Yet the results for the Medium Sample with 1,175 instead of 2,349 hedge funds hardly change. The error for relative alpha remains unchanged and is still insignificant, the error for absolute alpha increases to -0.0043 and has a p-value of 0.00. Only when we move to the Small Sample with 294 hedge funds widens the error for relative alpha sufficiently to -0.0020 to turn significant with a p-value of 0.06. The error for absolute alpha increases in magnitude to -0.0055 and has a p-value of 0.00. We conclude that a smaller cross-section does indeed worsen the performance of relative alpha but for that to happen, we need to reduce the number of potential peers by 87%.

Varying the length of the time series

Now, starting again from the Full Sample with 36 months of returns, we change the number of returns available. From Table VIII, Panel B, Short Sample, we see that reducing the time series to 24 months increases in magnitude the error for relative alpha very little from -0.0014 to a value of, again insignificant, -0.0015, while the error for absolute alpha increases in magnitude from -0.0036 to -0.0051, remaining strongly significant. Going to the Very Short Sample with only 12 returns, the error for relative alpha decreases to -0.0008 but is now marginally significant with a p-value of 0.07. The absolute alpha error turns positive with a value of 0.0019 and remains strongly significant. We find that relative alpha copes quite well with shorter time series, while absolute alpha performs worse with significant errors throughout.

We conclude from our simulation that the good performance of relative alpha is due to its capability of dealing with omitted factors. In order to achieve that feat, the method needs reasonably long time series of returns (24 months seem to suffice in our simulation) and a large enough cross section of peer funds (some 1000 seem to suffice), which can cancel out the unknown factor loadings.

V. Conclusion

We propose a novel performance measure, relative alpha, which assesses the outperformance of a hedge fund with respect to a group of peers. It exhibits the intriguing property that (potentially unknown) factors and their loadings partially cancels, as the peers are selected based on the variances of differences in returns, with more weight going to funds with low variances. We argue carefully, based on a theoretical model, a simulation study, and empirical evidence from hedge fund holdings reported on forms 13f, that this variance indeed leads to the desired cancellation. We are thus left with a measure of a hedge fund's weighted alpha difference with its peers. A nice side effect is that the investor does not even need to know the exact factor structure, nor any of the omitted factors.

Relative alpha beats absolute alpha in three areas: Explaining hedge fund returns in-sample, predicting future returns, and persistence of alpha. In terms of explaining returns, we find that using the seven closest peers identified by relative alpha explains more return variation (adjusted R^2 of 67% to 82%) than the seven factor Fung and Hsieh (2001) model (22% to 30%) and a similar result holds for the ten factor Namvar et al. (2016) model. In terms of predicting future returns, we sort hedge funds based on alpha and investigate the out-of-sample performance of the top, bottom, and top-minus-bottom deciles. Relative alpha deciles perform significantly better than absolute alpha deciles, where we use a number of performance measures (mean, variance, Sharpe ratio, manipulation-proof performance measure, Value at Risk, and second order stochastic dominance). Using alternative criteria for sorting (alphas of alternative factor models, the strategic distinctiveness index, or the manipulation-proof performance measure) does not change the results. The basic results are robust to changes in the sample and the methodology. Relative alpha even works for mutual funds, for which the Fama and French (1992) model is less prone to omitted factors than the Fung and Hsieh (2001) model for hedge fund returns. In terms of persistence of alpha, we use past alpha to predict future alpha. Relative alpha is strongly persistent in our sample, as opposed to absolute alpha, which is either insignificant or significantly anti-persistent (i.e., high absolute alpha in one period leads to low absolute alpha in the next period).

In a simulation study that realistically models omitted variables, we show that relative alpha works better than absolute alpha, if there is a large number of hedge funds (greater 1,000) in the cross-section and if time series of returns have a reasonable length (24 months or more).

Appendix

Appendix A. Proofs

We show that $Cov\left[Var[r_{i,t}-r_{j,t}], \sum_{l=1}^{L}(\beta_{i,l}-\beta_{j,l})^2 E[F_{l,t}]^2\right]$
$= (3-1)(2\sigma_\eta^2)^2 \sum_{l=1}^{L} E[F_l]^2 Var[F_l] > 0.$

As we repeatedly need the term $(\beta_{i,l}-\beta_{j,l})$, we define $z_l = \beta_{i,l}-\beta_{j,l} = \beta_l + \eta_{i,l} - \beta_l - \eta_{j,l} = \eta_{i,l} - \eta_{j,l}$ with distribution $\mathcal{N}(0, 2\sigma_\eta^2)$. We can now prove proposition 2:

$$
\begin{aligned}
&Cov\left[Var[r_{it}-r_{jt}], \sum_{l=1}^{L}(\beta_{i,l}-\beta_{j,l})^2 E[F_{l,t}]^2\right] \\
&= Cov\left[\sum_{l=1}^{L} z_l^2 Var[F_l] + \sum_{l=1}^{L}\sum_{k=1,k\neq l}^{L} z_l z_k Cov[F_l, F_k] + \sigma_i^2 + \sigma_j^2, \sum_{l=1}^{L} z_l^2 E[F_l]^2\right] \\
&= E\left[\sum_{l=1}^{L} z_l^4 E[F_l]^2 Var[F_l] + \sum_{l=1}^{L}\sum_{k=1,k\neq l}^{L} z_l^2 z_k^2 E[F_l]^2 Var[F_k] + \sum_{l=1}^{L} z_l^2 E[F_l]^2 \sigma_i^2 + \sum_{l=1}^{L} z_l^2 E[F_l]^2 \sigma_j^2\right] \\
&- E\left[\sum_{l=1}^{L} z_l^2 Var[F_l] + \sum_{l=1}^{L}\sum_{k=1,k\neq l}^{L} z_l z_k Cov[F_l, F_k] + \sigma_i^2 + \sigma_j^2\right] E\left[\sum_{l=1}^{L} z_l^2 E[F_l]^2\right] \\
&= 3(2\sigma_\eta^2)\sum_{l=1}^{L} E[F_l]^2 Var[F_l] + \sum_{l=1}^{L}\sum_{k=1,k\neq l}^{L} (2\sigma_\eta^2)(2\sigma_\eta^2) E[F_l]^2 Var[F_k] \\
&+ L2\sigma_\eta^2 E[F_l]^2 \sigma_i^2 + 2L\sigma_\eta^2 E[F_l]^2 \sigma_j^2 - \left[2\sigma_\eta^2 \sum_{l=1}^{L} Var[F_l] + \sigma_i^2 + \sigma_j^2\right]\left[L2\sigma_\eta^2 E[F_l]^2\right] \\
&= (3-1)(2\sigma_\eta^2)^2 \sum_{l=1}^{L} E[F_l]^2 Var[F_l] > 0
\end{aligned}
$$

Appendix B. Performance Measures

Manipulation-proof Performance Measure of Goetzmann et al. (2007)

$$\widehat{\Theta} = \frac{1}{(1-\rho)\,\triangle t} ln \left(\frac{1}{T} \sum_{t=1}^{T} \left[\frac{1+r_t}{1+r_{ft}} \right]^{1-\rho} \right), \tag{12}$$

where T is the total number of observations, $\triangle t$ is the length of time between observations, r_t is the return of a hedge fund in t, r_{ft} is the risk-free rate, ρ is the relative risk-aversion coefficient.

Strategy Distinctiveness Index of Sun et al. (2012)

$$SDI = 1 - corr(r_t, \mu), \tag{13}$$

where r_t is the return of a hedge fund in t, and μ is the average return of all funds belonging to the same style.

Appendix C. Factor Models

Fung and Hsieh (2001)

1. Bond Trend-Following Factor, lookback straddles
2. Currency Trend-Following Factor, lookback straddles
3. Commodity Trend-Following Factor, lookback straddles
4. Excess return on the S&P 500 index over the risk-free rate
5. Difference in the returns on the Wilshire Small Cap 1750 index and Wilshire Large Cap 750 index
6. The monthly change in the 10-year treasury constant maturity yield
7. The monthly change in the spread between Moody's Baa yield and 10-year treasury constant maturity yield

Agarwal and Naik (2004)

1. Returns on Russel 3000 Index

2. Returns on Morgan Stanely Capital International world excluding US Index
3. MSCI emerging market index
4. Salomon Brothers government and corporate bond index
5. Salomon Brothers world government bond
6. Lehman high yield index
7. Federal Reserve Bank competitiveness-weighted dollar index
8. Goldman Sachs commodity index
9. Factor-mimicking portfolio for size
10. Factor-mimicking portfolio for book-to-market equity
11. Factor-mimicking portfolio for the momentum effect
12. The monthly change in the spread between Moody's Baa yield and 10-year treasury constant maturity yield
13. At-the-money European call on the S&P 500 composite index
14. At-the-money European put on the S&P 500 composite index
15. Out-of-the-money European call on the S&P 500 composite index
16. Out-of-the-money European put on the S&P 500 composite index

Table I Descriptive statistics

The summary statistics are the equally weighted cross-sectional averages, standard deviations, minima, maxima, medians, 25% and 75% quantiles of: the mean monthly return, μ; the standard deviation of monthly returns, σ; the skewness, Skewness; the excess kurtosis, Kurtosis; the minimum, Minimum; the maximum, Maximum; the Sharpe ratio, Sharpe ratio; the Manipulation Proof Performance Measure (MPPM); and Value-at-Risk (VaR). Panel A represents statistics for hedge fund returns, Panel B represents statistics for mutual fund returns. The sample is February 1994 to June 2011.

Panel A: Hedge Funds

	Mean	Std.dev	Minimum	Maximum	Median	25% quantile	75% quantile
μ	0.0081	0.0033	-0.0037	0.0199	0.0076	0.0062	0.0095
σ	0.0391	0.0272	0.0021	0.1357	0.0335	0.0193	0.0491
Skewness	-0.4134	1.2978	-7.822	3.8999	-0.2271	-0.8562	0.3702
Kurtosis	8.6664	8.7884	2.6535	88.3032	6.0398	4.5052	8.3633
Minimum	-0.1477	0.0976	-0.5162	-0.0057	-0.1157	-0.1855	-0.0831
Maximum	0.1652	0.1531	0.0094	1.0106	0.1245	0.0682	0.2081
Sharpe ratio	0.1831	0.1076	-0.2072	0.7654	0.1672	0.1184	0.2338
MPPM	0.0273	0.0531	-0.2074	0.1368	0.0394	0.0169	0.0535
VaR	-0.0528	0.0398	-0.1977	-0.0009	-0.0442	-0.0692	-0.0239

Panel B: Mutual Funds

	Mean	Std.dev	Minimum	Maximum	Median	25% quantile	75% quantile
μ	0.0057	0.0022	-0.0055	0.0172	0.0054	0.004	0.0072
σ	0.0385	0.0212	0.0017	0.1374	0.0397	0.0176	0.0532
Skewness	-0.5214	0.4968	-5.1031	1.8331	-0.5512	-0.7051	-0.2654
Kurtosis	5.4357	3.4634	1.3254	54.1807	4.5841	4.0418	5.5513
Minimum	-0.1514	0.0854	-0.8399	0.0001	-0.1597	-0.211	-0.0758
Maximum	0.1236	0.0776	0.0047	0.6897	0.1131	0.0689	0.1573
Sharpe ratio	0.0928	0.0666	-1.2323	0.8543	0.0924	0.0614	0.1212
MPPM	0.0018	0.0351	-0.9734	0.0867	0.0099	-0.006	0.0194
VaR	-0.0612	0.036	-0.1979	0.0002	-0.0632	-0.0883	-0.0247

Table II Current hedge fund performance

The table shows the in-sample performance of factor models using the seven and ten closest peers identified by the distance measure proposed in the paper as pseude-factors (7 Peers and 10 Peers), the Fung and Hsieh (2001) (FH) model, and the Namvar et al. (2016) (NPPR) model. We use rolling windows of 36 months (Panel A), 24 months (Panel B), and 12 months (Panel C). The table reports average alpha estimates (Alpha), average standard errors (Std. dev.), and average adjusted R^2.

Panel A: 36 months in-sample

	7 Peers	10 Peers	FH	NPPR
Alpha	-0.0029	-0.0037	0.0063	0.0072
Std. dev.	0.0061	0.0083	0.0055	0.0059
adj. R^2	0.6797	0.6734	0.2977	0.2388

Panel B: 24 months in-sample

	7 Peers	10 Peers	FH	NPPR
Alpha	-0.0036	-0.0048	0.0064	0.0068
Std. dev.	0.0058	0.0074	0.0069	0.0078
adj. R^2	0.6672	0.6769	0.2840	0.2353

Panel C: 12 months in-sample

	7 Peers	10 Peers	FH	NPPR
Alpha	-0.0039	-0.0050	0.0065	0.0054
Std. dev.	0.0093	0.0209	0.0138	0.0206
adj. R^2	0.8229	0.7768	0.2165	0.2199

Table III Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds

The table demonstrates the out-of-sample performance of top, bottom, and top-minus-bottom decile portfolios constructed by sorting hedge funds based on relative alpha (REL) and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test.

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0107	0.0100	0.0027	0.0039	0.0080	0.0061
p-value	1.0000	0.7438	1.0000	0.6471	1.0000	0.2804
σ	0.0208	0.0247	0.0214	0.0285	0.0099	0.0228
p-value	1.0000	0.0149	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.4182	0.3207	0.0316	0.0641	0.6073	0.1768
p-value	1.0000	0.0089	1.0000	0.3988	1.0000	0.0001
MPPM	0.0959	0.0839	-0.0002	0.0071	0.0693	0.0391
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0140	-0.0217	-0.0213	-0.0318	-0.0021	-0.0209
p-value	1.0000	0.0100	1.0000	0.0140	1.0000	0.0000
SSD	1.0000	1.6867	1.0000	1.5453	1.0000	5.9536
p-value	1.0000	0.1160	1.0000	0.1320	1.0000	0.0000

Table IV Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with alternative measures

The table demonstrates out-of-sample performance characteristics of top (Panel A), bottom (Panel B), and top-minus-bottom (Panel C) decile portfolios constructed by sorting hedge funds based on relative alpha (REL), absolute alpha estimated from Agarwal and Naik (2004) (AN) and Namvar et al. (2016) (NPPR), the Strategy Distinctiveness Index from Sun et al. (2012) (SDI), and the Manipulation Proof Performance Measure from Goetzmann et al. (2007) (MPPM) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test.

Panel A: Top Portfolio

	REL	AN	NPPR	SDI	MPPM
μ	0.0107	0.0093	0.0103	0.0064	0.0107
p-value	1.0000	0.3736	0.8747	0.0474	0.9967
σ	0.0208	0.0217	0.0289	0.0225	0.0278
p-value	1.0000	0.8328	0.0000	0.2639	0.0001
Sharpe ratio	0.4182	0.3026	0.2865	0.1944	0.3140
p-value	1.0000	0.1461	0.0296	0.0000	0.0654
MPPM	0.0959	0.0701	0.0843	0.0432	0.0903
p-value	1.0000	0.0000	0.0000	0.0000	0.0000
VaR	-0.0140	-0.0174	-0.0243	-0.0204	-0.0235
p-value	1.0000	0.2600	0.0040	0.0020	0.0100
SSD	1.0000	0.8941	2.4108	2.1604	1.8292
p-value	1.0000	0.3420	0.0180	0.0330	0.0870

Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with alternative measures (cont'd)

Panel B: Bottom Portfolio

	REL	AN	NPPR	SDI	MPPM
μ	0.0027	0.0057	0.0040	0.0061	0.0044
p-value	1.0000	0.3843	0.5733	0.0881	0.5458
σ	0.0214	0.0323	0.0244	0.0185	0.0315
p-value	1.0000	0.0000	0.0629	0.0393	0.0000
Sharpe ratio	0.0316	0.0920	0.0808	0.2218	0.0728
p-value	1.0000	0.0610	0.3078	0.0002	0.2858
MPPM	-0.0002	0.0164	0.0128	0.0427	0.0095
p-value	1.0000	0.0000	0.0000	0.0000	0.0000
VaR	-0.0213	-0.0300	-0.0219	-0.0150	-0.0247
p-value	1.0000	0.1000	0.8780	0.0000	0.2220
SSD	1.0000	1.3704	0.2180	-1.2324	1.3105
p-value	1.0000	0.1400	0.5990	0.9650	0.1470

Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with alternative measures (cont'd)

Panel C: Top-Bottom Portfolio

	REL	AN	NPPR	SDI	MPPM
μ	0.0080	0.0036	0.0063	0.0003	0.0064
p-value	1.0000	0.0277	0.4308	0.0000	0.5004
σ	0.0099	0.0258	0.0286	0.0092	0.0324
p-value	1.0000	0.0000	0.0000	0.2705	0.0000
Sharpe ratio	0.6073	0.0341	0.1482	-0.1885	0.1341
p-value	1.0000	0.0001	0.0002	0.0000	0.0000
MPPM	0.0693	-0.0009	0.0366	-0.0227	0.0333
p-value	1.0000	0.0000	0.0000	0.0000	0.0000
VaR	-0.0021	-0.0269	-0.0272	-0.0098	-0.0346
p-value	1.0000	0.0000	0.0000	0.0000	0.0000
SSD	1.0000	5.8580	6.1819	9.0194	6.3337
p-value	1.0000	0.0000	0.0000	0.0000	0.0000

Table V Predicting future portfolio performance within self-reported styles: relative alpha versus absolute alpha

The table demonstrates out-of-sample performance of top, bottom, and top-minus-bottom decile portfolios constructed by sorting hedge funds based on relative alpha (REL) and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the three largest self-reported styles: global macro (16% of the sample in Panel A), equity long-short (49% in Panel B), and relative value (10% in Panel C). We use the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test.

Panel A: Global macro

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0091	0.0083	0.0034	0.0049	0.0057	0.0034
p-value	1.0000	0.7341	1.0000	0.4668	1.0000	0.2803
σ	0.0201	0.0255	0.0181	0.0237	0.0144	0.0266
p-value	1.0000	0.0008	1.0000	0.0001	1.0000	0.0000
Sharpe ratio	0.3526	0.2460	0.0736	0.1211	0.2518	0.0495
p-value	1.0000	0.0530	1.0000	0.3704	1.0000	0.0289
MPPM	0.0768	0.0633	0.0102	0.0246	0.0398	0.0028
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0128	-0.0203	-0.0179	-0.0178	-0.0128	-0.0289
p-value	1.0000	0.0000	1.0000	0.9660	1.0000	0.0000
SSD	1.0000	2.3237	1.0000	1.8597	1.0000	4.1962
p-value	1.0000	0.0180	1.0000	0.0810	1.0000	0.0000

Predicting future portfolio performance within self-reported styles: relative alpha versus absolute alpha (cont'd)

Panel B: Equity long short

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0114	0.0105	0.0025	0.0038	0.0089	0.0067
p-value	1.0000	0.7829	1.0000	0.7180	1.0000	0.4213
σ	0.0317	0.0330	0.0282	0.0408	0.0155	0.0346
p-value	1.0000	0.5678	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.2959	0.2562	0.0164	0.0423	0.4461	0.1349
p-value	1.0000	0.3218	1.0000	0.8536	1.0000	0.0020
MPPM	0.0940	0.0818	-0.0089	-0.0099	0.0774	0.0348
p-value	1.0000	0.0000	1.0000	0.3174	1.0000	0.0000
VaR	-0.0300	-0.0326	-0.0337	-0.0469	-0.0063	-0.0267
p-value	1.0000	0.3740	1.0000	0.0180	1.0000	0.0000
SSD	1.0000	0.6390	1.0000	1.8081	1.0000	4.1984
p-value	1.0000	0.4150	1.0000	0.0800	1.0000	0.0000

Predicting future portfolio performance within self-reported styles: relative alpha versus absolute alpha (cont'd)

Panel C: Relative value

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0111	0.0107	0.0039	0.0031	0.0072	0.0076
p-value	1.0000	0.8012	1.0000	0.6571	1.0000	0.7960
σ	0.0160	0.0154	0.0153	0.0181	0.0122	0.0147
p-value	1.0000	0.0315	1.0000	0.0194	1.0000	0.0091
Sharpe ratio	0.5596	0.5564	0.1181	0.0598	0.4163	0.3685
p-value	1.0000	0.9534	1.0000	0.3010	1.0000	0.6503
MPPM	0.1032	0.0988	0.0175	0.0069	0.0590	0.0618
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0068	-0.0077	-0.0119	-0.0162	-0.0079	-0.0106
p-value	1.0000	0.6820	1.0000	0.0280	1.0000	0.6200
SSD	1.0000	0.2623	1.0000	1.1778	1.0000	1.5202
p-value	1.0000	0.5530	1.0000	0.2190	1.0000	0.1380

Table VI Predicting future portfolio performance: relative alpha versus absolute alpha. Mutual funds

The table demonstrates out-of-sample performance of top, bottom, and top-minus-bottom decile portfolios constructed by sorting mutual funds based on relative alpha and the absolute alpha estimated from Fama and French (1992) (FF) for the full sample February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test.

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FF	REL	FF	REL	FF
μ	0.0067	0.0060	0.0037	0.0048	0.0030	0.0012
p-value	1.0000	0.8860	1.0000	0.8151	1.0000	0.5022
σ	0.0455	0.0536	0.0415	0.0472	0.0112	0.0352
p-value	1.0000	0.0245	1.0000	0.0765	1.0000	0.0000
Sharpe ratio	0.0998	0.0709	0.0364	0.0547	0.0728	-0.0277
p-value	1.0000	0.2057	1.0000	0.6458	1.0000	0.2727
MPPM	0.0156	-0.0095	-0.0142	-0.0104	0.0077	-0.0341
p-value	1.0000	0.0000	1.0000	0.0355	1.0000	0.0000
VaR	-0.0524	-0.0646	-0.0515	-0.0593	-0.0093	-0.0425
p-value	1.0000	0.0900	1.0000	0.2260	1.0000	0.0000
SSD	1.0000	1.0217	1.0000	0.7139	1.0000	6.3038
p-value	1.0000	0.2780	1.0000	0.3990	1.0000	0.0000

Table VII Persistence of alpha: relative alpha versus absolute alpha

The table presents estimated slope coefficients (b) from stacked, non-overlapping linear regressions: $\triangle_{2i} = a_\Delta + b_\Delta \triangle_{1i} + \omega_i$ for relative alpha (REL) and $\alpha_{2i} = a_\alpha + b_\alpha \alpha_{1i} + v_i$ for absolute alpha estimated from the Fung and Hsieh (2001) (FH) or the Namvar et al. (2016) (NPPR) model. It also provides Newey-West t-statistics on the significance of the b estimates as well as adjusted R^2. By stacking the regressions, we assume that the slope coefficients are constant across periods. We consider three cases where formation and evaluation periods each consist of 12 months (Panel A), 24 months (Panel B), and 36 months (Panel C).

Panel A: 12 months in-sample (12 months out-of-sample)

	REL	FH	NPPR
a	-0.0057	0.0050	0.0066
t-stat	-10.7852	17.0346	13.6835
b	0.6543	-0.0311	-0.0847
t-stat	24.0326	-2.4518	-2.6462
adj. R^2	0.2351	0.0000	0.0030

Persistence of alpha: relative alpha versus absolute alpha (cont'd)

Panel B: 24 months in-sample (24 months out-of-sample)

	REL	FH	NPPR
a	-0.0005	0.0054	0.0042
t-stat	-6.4940	24.7098	3.4867
b	0.6877	-0.0440	0.1310
t-stat	16.4548	-2.9289	1.1034
R^2	0.2903	0.0018	0.0022

Panel C: 36 months in-sample (36 months out-of-sample)

	REL	FH	NPPR
a	-0.0003	0.0049	0.0066
t-stat	-3.5808	21.2341	19.3733
b	0.6708	-0.0104	-0.0703
t-stat	13.4251	-0.6324	-2.5024
R^2	0.2731	0.0000	0.0042

Table VIII Simulation study

The table demonstrates results of the simulation study as described in Section IV. We simulate hedge fund returns by using a factor model that we assume to be true, using 100 simulation runs. In order to preserve the empirical characteristics of our hedge fund returns, we extract the first 30 principal components and set the assumed true parameters equal to the estimated parameters as observed in the data. The cross-sectional dimension in the base case is denoted as N=2,349 and the length of the time series is T=36. In Panel A we keep T constant but vary N. In Panel B, we keep N constant but vary T. In the first row we report the true relative alpha (REL) based on Equation (11), where we substitute $E[r_{it} - r_{jt}]$ with the differences in true alphas, and the true absolute alpha (ABS). In the second row of results, we estimate relative alpha as in Equation (8) and absolute alpha based on the Fung and Hsieh (2001) model. In the third row we report the differences between the true parameters and the estimated ones. The fourth row (p-value) contains the p-values of testing the differences against zero with a t-test.

Panel A: Variation in the Size of the Cross-Section

	Full sample N=2,349		Medium sample N=1,175		Small sample N=294	
	REL	ABS	REL	ABS	REL	ABS
Average true alpha	-0.0005	0.0015	-0.0004	0.0011	-0.0007	0.0000
Average estimated alpha	0.0009	0.0051	0.0010	0.0054	0.0013	0.0055
Difference	-0.0014	-0.0036	-0.0014	-0.0043	-0.0020	-0.0055
p-value	0.3022	0.0000	0.8614	0.0000	0.0574	0.0000

Simulation study (cont'd)

Panel B: Variation in Time-Series

	Full sample T=36		Short sample T=24		Very short sample T=12	
	Rel	Abs	Rel	Abs	Rel	Abs
Average true alpha	-0.0005	0.0015	-0.0005	0.0015	-0.0005	0.0015
Average estimated alpha	0.0009	0.0051	0.0010	0.0066	0.0003	-0.0004
Difference	-0.0014	-0.0036	-0.0015	-0.0051	-0.0008	0.0019
p-value	0.3022	0.0000	0.3899	0.0000	0.0692	0.0033

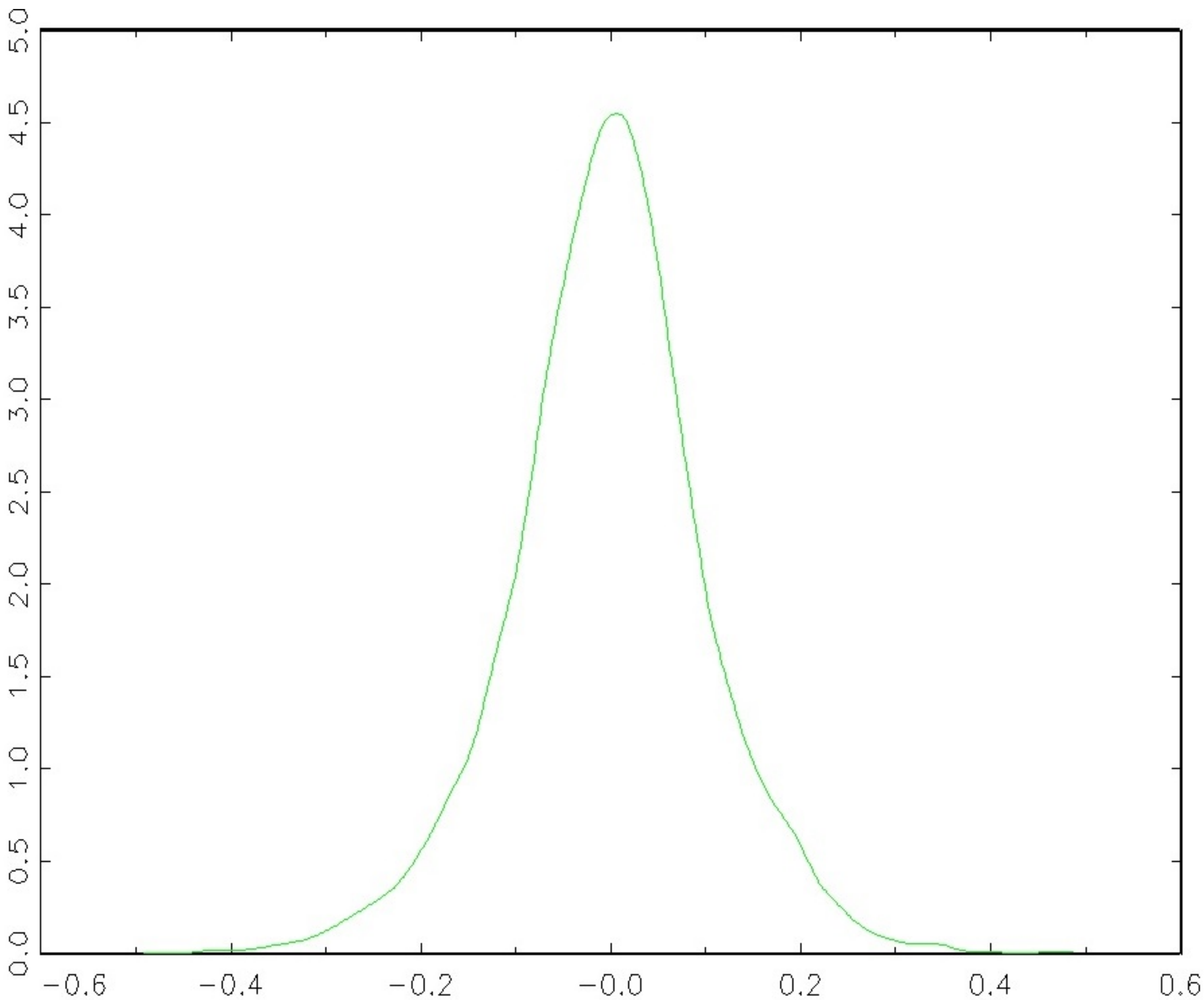

Figure 1. Density of Approximation Errors based on hedge fund holdings in 13f filings. The figure shows the density of the approximation errors $\sum_{l=1}^{L}(\beta_{i,l} - \beta_{j,l})E[F_{l,t}]$, where we treat each security reported in the 13f filings as a factor and use the value-based weight as its beta.

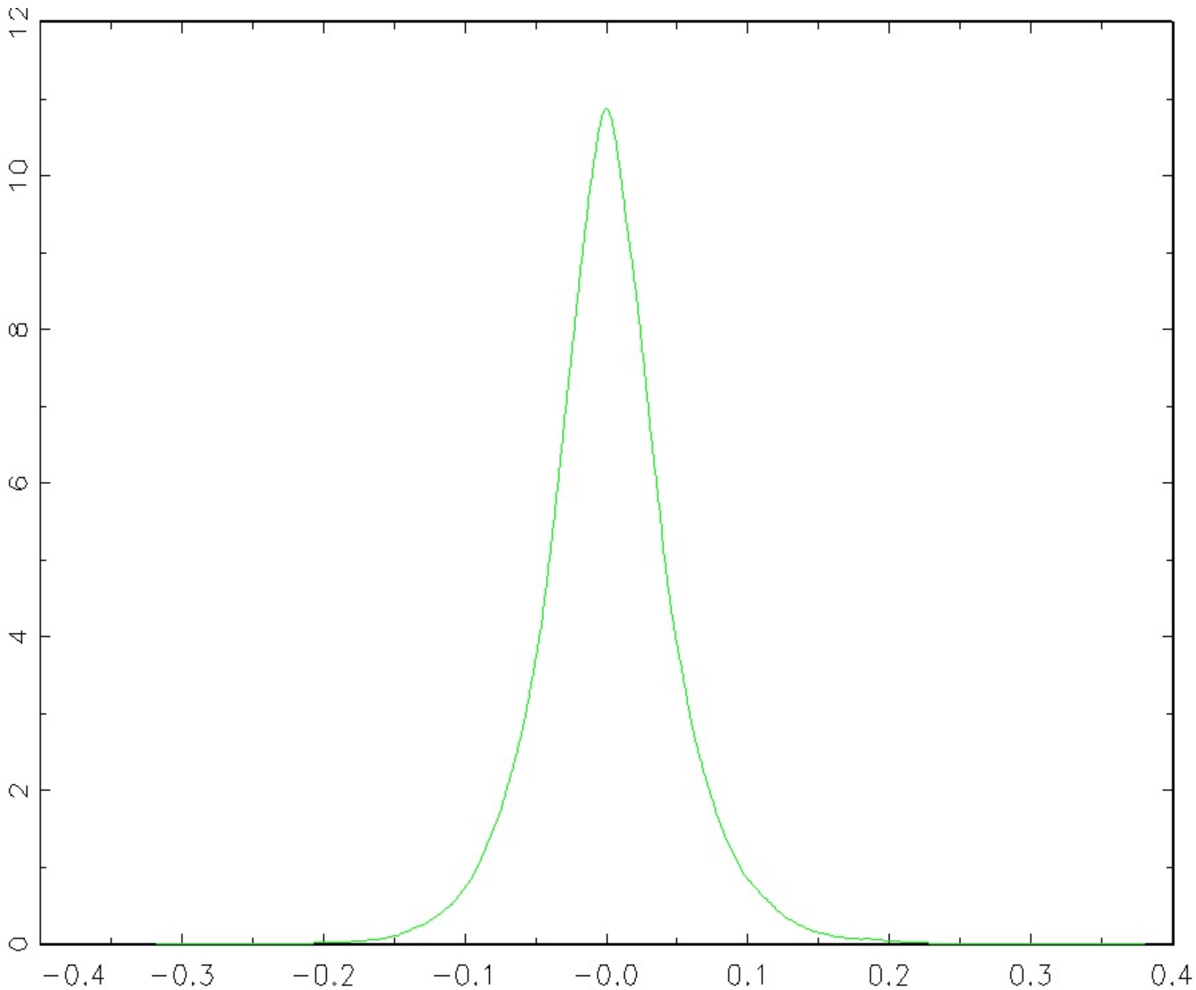

Figure 2. Density of approximation errors based on simulated hedge fund returns. The figure shows the density of the approximation errors $\sum_{l=1}^{L}(\beta_{i,l} - \beta_{j,l})E[F_{l,t}]$, where we use simulated hedge fund returns and risk factors to obtain β and $E[F_{l,t}]$.

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Internet Appendix for "Relative Alpha"

Jens Carsten Jackwerth

Anna Slavutskaya

Table IA.1 Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with 24 month in-sample estimation

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting hedge funds based on relative alpha and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test. For the rolling window estimations we use 24 months.

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0116	0.0114	0.0043	0.0040	0.0073	0.0073
p-value	1.0000	0.9371	1.0000	0.9206	1.0000	0.9711
σ	0.0211	0.0264	0.0278	0.0261	0.0198	0.0241
p-value	1.0000	0.0011	1.0000	0.3617	1.0000	0.0042
Sharpe ratio	0.4463	0.3490	0.0761	0.0710	0.2556	0.2136
p-value	1.0000	0.0259	1.0000	0.9280	1.0000	0.8228
MPPM	0.1039	0.0975	0.0131	0.0099	0.0514	0.0515
p-value	1.0000	0.0000	1.0000	0.0018	1.0000	0.3287
VaR	-0.0142	-0.0222	-0.0210	-0.0253	-0.0032	-0.0213
p-value	1.0000	0.0480	1.0000	0.0100	1.0000	0.0000
SSD	1.0000	1.6893	1.0000	1.6043	1.0000	2.5050
p-value	1.0000	0.1020	1.0000	0.1240	1.0000	0.0170

Table IA.2 Predicting future portfolio performance: relative alpha versus absolute alpha. Correcting for the boundary bias of the kernel estimates.

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test. We correct for the boundary bias of the kernel estimates by using the locally weighted regression method proposed by Hastie and Loader (1993).

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0111	0.0100	0.0028	0.0039	0.0083	0.0061
p-value	1.0000	0.7438	1.0000	0.6471	1.0000	0.2804
σ	0.0210	0.0247	0.0211	0.0285	0.0098	0.0228
p-value	1.0000	0.0149	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.4189	0.3207	0.0315	0.0641	0.6109	0.1768
p-value	1.0000	0.0083	1.0000	0.3984	1.0000	0.0000
MPPM	0.0958	0.0839	-0.0004	0.0071	0.0713	0.0391
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0135	-0.0217	-0.0220	-0.0318	-0.0015	-0.0209
p-value	1.0000	0.0098	1.0000	0.0148	1.0000	0.0000
SSD	1.0000	1.6898	1.0000	1.5441	1.0000	5.9936
p-value	1.0000	0.1151	1.0000	0.1335	1.0000	0.0000

Table IA.3 Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with 12 month holding period

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting hedge funds based on relative alpha and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test. We hold hedge funds in our portfolios for the period of 12 months.

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH7	REL	FH	REL	FH7
μ	0.0084	0.0073	0.0040	0.0052	0.0044	0.0021
p-value	1.0000	0.6144	1.0000	0.6133	1.0000	0.1590
σ	0.0206	0.0252	0.0190	0.0268	0.0095	0.0214
p-value	1.0000	0.0051	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.3100	0.2076	0.1017	0.1158	0.2549	0.0023
p-value	1.0000	0.0040	1.0000	0.7840	1.0000	0.0469
MPPM	0.0687	0.0511	0.0168	0.0244	0.0269	-0.0076
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0181	-0.0248	-0.0194	-0.0259	-0.0064	-0.0239
p-value	1.0000	0.0080	1.0000	0.0120	1.0000	0.0000
SSD	1.0000	1.5608	1.0000	2.0958	1.0000	5.3898
p-value	1.0000	0.1390	1.0000	0.0510	1.0000	0.0000

Table IA.4 Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with de-smoothed returns

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test. We de-smooth hedge fund returns according to Getmansky, Lo, and Makarov (2004).

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH7	REL	FH	REL	FH7
μ	0.0089	0.0086	0.0038	0.0046	0.0051	0.0040
p-value	1.0000	0.9020	1.0000	0.7847	1.0000	0.6023
σ	0.0228	0.0254	0.0230	0.0372	0.0104	0.0292
p-value	1.0000	0.1215	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.3014	0.2583	0.0756	0.0695	0.2948	0.0661
p-value	1.0000	0.2404	1.0000	0.8448	1.0000	0.0116
MPPM	0.0729	0.0672	0.0113	0.0054	0.0349	0.0081
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0211	-0.0221	-0.0231	-0.0417	-0.0069	-0.0307
p-value	1.0000	0.3660	1.0000	0.0000	1.0000	0.0000
SSD	1.0000	1.0100	1.0000	2.7141	1.0000	5.7986
p-value	1.0000	0.2850	1.0000	0.0070	1.0000	0.0000

Table IA.5 Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds during crisis versus non-crisis period.

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting hedge funds based on relative alpha and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the crises periods (Panel A) and non-crises periods (Panel B) as defined by NBER classification. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test.

Panel A: Crisis Period

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0006	-0.0027	-0.0067	-0.0069	0.0072	0.0042
p-value	1.0000	0.6703	1.0000	0.9809	1.0000	0.5510
σ	0.0245	0.0315	0.0333	0.0336	0.0150	0.0219
p-value	1.0000	0.1930	1.0000	0.9601	1.0000	0.0499
Sharpe ratio	-0.0438	-0.1361	-0.2486	-0.2526	0.3714	0.1169
p-value	1.0000	0.0475	1.0000	0.9257	1.0000	0.3106
MPPM	-0.0235	-0.0693	-0.1199	-0.1230	0.0629	0.0224
p-value	1.0000	0.0000	1.0000	0.2859	1.0000	0.0000
VaR	-0.0351	-0.0485	-0.0477	-0.0566	-0.0102	-0.0291
p-value	1.0000	0.1700	1.0000	0.2960	1.0000	0.2020
SSD	1.0000	1.2589	1.0000	0.2461	1.0000	1.8221
p-value	1.0000	0.2120	1.0000	0.5760	1.0000	0.0720

Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds during crisis versus non-crisis period (cont'd).

Panel B: Non-crisis Period

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH	REL	FH	REL	FH
μ	0.0124	0.0120	0.0043	0.0056	0.0081	0.0064
p-value	1.0000	0.8812	1.0000	0.5842	1.0000	0.3607
σ	0.0198	0.0229	0.0184	0.0273	0.0089	0.0230
p-value	1.0000	0.0503	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.5213	0.4324	0.1151	0.1281	0.6893	0.1856
p-value	1.0000	0.0782	1.0000	0.7949	1.0000	0.0000
MPPM	0.1155	0.1091	0.0194	0.0285	0.0703	0.0418
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0101	-0.0162	-0.0194	-0.0281	-0.0014	-0.0160
p-value	1.0000	0.0260	1.0000	0.0020	1.0000	0.0000
SSD	1.0000	1.4120	1.0000	2.0243	1.0000	5.7738
p-value	1.0000	0.1710	1.0000	0.0440	1.0000	0.0000

Table IA.6 Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds open to new investments

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting hedge funds based on relative alpha and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test. We restrict the sample of hedge funds to the ones open to new investors.

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH7	REL	FH	REL	FH7
μ	0.0107	0.0096	0.0027	0.0035	0.0080	0.0062
p-value	1.0000	0.6587	1.0000	0.7706	1.0000	0.3485
σ	0.0218	0.0250	0.0216	0.0303	0.0104	0.0251
p-value	1.0000	0.0527	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.3952	0.3018	0.0296	0.0463	0.5746	0.1629
p-value	1.0000	0.0152	1.0000	0.6870	1.0000	0.0002
MPPM	0.0942	0.0792	-0.0008	0.0001	0.0685	0.0378
p-value	1.0000	0.0000	1.0000	0.4341	1.0000	0.0000
VaR	-0.0164	-0.0223	-0.0228	-0.0330	-0.0022	-0.0211
p-value	1.0000	0.0100	1.0000	0.0080	1.0000	0.0000
SSD	1.0000	1.4780	1.0000	1.8347	1.0000	5.8946
p-value	1.0000	0.1470	1.0000	0.0710	1.0000	0.0000

Table IA.7 Predicting future portfolio performance: relative alpha versus absolute alpha. Hedge funds with AUM> 20 mio

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha estimated from Fung and Hsieh (2001) (FH) for the full sample from February 1994 to June 2011. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test. We exclude any hedge funds with asset under management less than 20 Mio.

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FH7	REL	FH	REL	FH7
μ	0.0107	0.0100	0.0027	0.0039	0.0080	0.0061
p-value	1.0000	0.7438	1.0000	0.6471	1.0000	0.2804
σ	0.0208	0.0247	0.0214	0.0285	0.0099	0.0228
p-value	1.0000	0.0149	1.0000	0.0000	1.0000	0.0000
Sharpe ratio	0.4182	0.3207	0.0316	0.0641	0.6073	0.1768
p-value	1.0000	0.0089	1.0000	0.3988	1.0000	0.0001
MPPM	0.0959	0.0839	-0.0002	0.0071	0.0693	0.0391
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0140	-0.0217	-0.0213	-0.0318	-0.0021	-0.0209
p-value	1.0000	0.0060	1.0000	0.0060	1.0000	0.0000
SSD	1.0000	1.6867	1.0000	1.5453	1.0000	5.9536
p-value	1.0000	0.1050	1.0000	0.1270	1.0000	0.0000

Table IA.8 Predicting future portfolio performance: relative alpha versus absolute alpha. Mutual funds during crisis versus non-crisis period.

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting mutual funds based on relative alpha and absolute alpha estimated from Fama and French (1992) (FF) for the crises periods (Panel A) and non-crises periods (Panel B) as defined by NBER classification. The characteristics include the monthly mean (μ), standard deviation (σ), Sharpe ratio, Manipulation Proof Performance Measure (MPPM), and Value-at-Risk (VaR). We also provide p-values for differences between the mean, standard deviaton, Sharpe Ratio, MPPM, and VaR of the relative alpha portfolios and their competitors. The last two rows provide t-statistics and p-values of the Davidson and Duclos (2013) second-order stochastic non-dominance test.

Panel A: Crisis period

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FF	REL	FF	REL	FF
μ	-0.0137	-0.0181	-0.0140	-0.0098	0.0003	-0.0083
p-value	1.0000	0.8353	1.0000	0.7954	1.0000	0.3452
σ	0.0676	0.0896	0.0655	0.0555	0.0124	0.0462
p-value	1.0000	0.1419	1.0000	0.3878	1.0000	0.0000
Sharpe ratio	-0.2259	-0.2199	-0.2383	-0.2051	-0.1037	-0.2147
p-value	1.0000	0.7996	1.0000	0.7967	1.0000	0.7451
MPPM	-0.2710	-0.3936	-0.2694	-0.1940	-0.0183	-0.1584
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.1078	-0.1322	-0.0989	-0.0837	-0.0200	-0.0832
p-value	1.0000	0.0040	1.0000	0.2420	1.0000	0.0000
SSD	1.0000	0.9476	1.0000	-0.2654	1.0000	3.4122
p-value	1.0000	0.3270	1.0000	0.7800	1.0000	0.0010

Predicting future portfolio performance: relative alpha versus absolute alpha. Mutual funds during crisis versus non-crisis period (cont'd).

Panel B: Non-crisis period

	Top Portfolio		Bottom Portfolio		Top-Bottom Portfolio	
	REL	FF	REL	FF	REL	FF
μ	0.0103	0.0102	0.0068	0.0073	0.0035	0.0029
p-value	1.0000	0.9848	1.0000	0.9086	1.0000	0.8235
σ	0.0396	0.0436	0.0351	0.0453	0.0109	0.0328
p-value	1.0000	0.2322	1.0000	0.0013	1.0000	0.0000
Sharpe ratio	0.2019	0.1815	0.1285	0.1109	0.1074	0.0181
p-value	1.0000	0.4722	1.0000	0.6486	1.0000	0.3248
MPPM	0.0672	0.0604	0.0316	0.0223	0.0123	-0.0121
p-value	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
VaR	-0.0425	-0.0424	-0.0350	-0.0510	-0.0081	-0.0364
p-value	1.0000	0.6880	1.0000	0.0980	1.0000	0.0000
SSD	1.0000	0.6236	1.0000	1.4956	1.0000	5.5487
p-value	1.0000	0.3960	1.0000	0.1310	1.0000	0.0000