# Online Appendix to Price Points and Price Dynamics

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**Abstract**

This document derives expressions for the optimal prices in the models introduced in the main paper. In addition, it contains the derivations of the corresponding Phillips curves. Finally, it includes a discussion of the relationship between the PPSI model and menu-cost models.

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# A Optimal Price-setting and the Phillips Curve in the PPSI Model and the PP Model

## A.1 Price setting in the PPSI model and the PP model

In this appendix, we determine firms' optimal price-setting behavior in the PPSI model and the PP model. Equation (4) and the demand function (1) can be used to write profits (3) as

$$\Pi_{j,t} = \left(\frac{Q_{j,t}}{P_t}\right)^{1-\varepsilon} Y_t - \frac{W_t}{P_t} A_t^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} \left(\frac{Q_{j,t}}{P_t}\right)^{-\frac{\varepsilon}{\gamma}} Y_t^{\frac{1}{\gamma}}. \tag{26}$$

The respective first-order condition is

$$(1-\varepsilon)\left(\frac{Q^*_{j,t}}{P_t}\right)^{-\varepsilon} Y_t + \frac{\varepsilon}{\gamma}\frac{W_t}{P_t} A_t^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} \left(\frac{Q^*_{j,t}}{P_t}\right)^{-\frac{\varepsilon}{\gamma}-1} Y_t^{\frac{1}{\gamma}} = 0, \tag{27}$$

from which we obtain the optimal price $Q^*_{j,t}$ that would be chosen if there were no information frictions and if prices were not restricted to the set of price points:

$$\left(\frac{Q^*_{j,t}}{P_t}\right)^{\frac{\gamma+\varepsilon(1-\gamma)}{\gamma}} = \frac{\varepsilon}{(\varepsilon-1)\gamma}\frac{W_t}{P_t} A_t^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} Y_t^{\frac{1-\gamma}{\gamma}} \tag{28}$$

A log-linear approximation of this condition yields the following hypothetical optimal log price in the absence of a PP restriction and information rigidities:

$$q^*_{j,t} = \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\left[-\frac{1}{\gamma}x_{j,t} + w_t - p_t - \ln\left(\frac{\overline{W}}{P}\right) + \frac{1-\gamma}{\gamma}\hat{Y}_t - \frac{1}{\gamma}\hat{A}_t\right] + p_t \tag{29}$$

We observe that the expression $w_t - p_t - \ln\left(\frac{\overline{W}}{P}\right) + \frac{1-\gamma}{\gamma}\hat{Y}_t - \frac{1}{\gamma}\hat{A}_t$ represents the deviation of aggregate unit labor costs from its steady-state value in this economy, $\widehat{ulc}_t = w_t - p_t - \ln\left(\frac{\overline{W}}{P}\right) + \hat{N}_t - \hat{Y}_t$, since the log-linearized aggregate production function is given by $\hat{N}_t = \frac{1}{\gamma}\hat{Y}_t - \frac{1}{\gamma}\hat{A}_t$.[1] Therefore the hypothetical optimal log price in the absence of a PP restriction

[1] In general, price dispersion $s_t = \int_0^1 \left(\frac{Q_{j,t}}{P_t}\right)^{-\varepsilon/\gamma}(X_{j,t})^{-1/\gamma}dj$ affects the relationship between employment and output, as $Y_t = (A_t N_t^{\gamma})/s_t$ (see Ascari and Sbordone (2014)). In the PP model, $s_t$ is always constant. In the PPSI model, $s_t$ reaches a minimum in the steady state, which means that small perturbations have no first-order effect on $s_t$. Hence $\hat{Y}_t = \hat{A}_t + \gamma\hat{N}_t$ holds approximately in both cases. The same result does not hold under Calvo pricing when the steady-state inflation rate is positive.

and information frictions is given by

$$q_{j,t}^{*} = \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left[ -\frac{1}{\gamma} x_{j,t} + \widehat{ulc}_t \right] + p_t. \tag{30}$$

In the following, we analyze the optimal price setting behavior under the assumption that firm $j$ can only select price points. Consider a quadratic approximation of the profit function around its maximum. Then firm $j$'s profit-maximizing admissible log price $q_{j,t}$ is the element in the set $\Delta_j \cdot \mathbb{Z} - u_j$ that is closest to $q_{j,t}$. Thus the optimal log price in the PP model is

$$q_{j,t}^{PP} = \mathcal{T}_j \left\{ \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left( -\frac{1}{\gamma} x_{j,t} + \widehat{ulc}_t \right) + p_t \right\}, \tag{31}$$

where $\mathcal{T}_j : \mathbb{R} \to \mathbb{R}$ has been defined in the main text as an operator that maps the hypothetical, optimal log price of producer $j$ in the absence of the PP restriction, $q_{j,t}^{*}$, to the closest corresponding log price point $q_{j,t}^{PP} \in \Delta_j \cdot \mathbb{Z} - u_j$.

Optimal price-setting in the PPSI model can be determined in an analogous way. Given that firm $j$ has last updated its information in period $t-i$, its profit-maximizing admissible log price $q_{j,t}$ is the element in the set $\Delta_j \cdot \mathbb{Z} - u_j$ that is closest to $\mathbb{E}_{t-i}[q_{j,t}]$. Hence a firm $j$ that has received new information $i$ periods ago selects

$$q_{j,t}^{PPSI} = \mathcal{T}_j \left\{ \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left( -\frac{1}{\gamma} x_{j,t} + \mathbb{E}_{t-i} \left[ \widehat{ulc}_t \right] \right) + \mathbb{E}_{t-i} \left[ p_t \right] \right\}. \tag{32}$$

Note that we have used the fact that $x_{j,t}$ does not change in periods where the firm is not subject to idiosyncratic shocks, i.e. $\mathbb{E}_{t-i} \left[ x_{j,t} \right] = x_{j,t}$. $\square$

## A.2 The Phillips curve for the PPSI and the PP model

In the PP model, the individual differences of firms' log prices from $q_{j,t}^{*}$ and the individual productivities $x_{j,t}$ wash out in the aggregation when the price level is computed from individual prices.[2] As a consequence, $\widehat{ulc}_t = 0$ holds or, equivalently,

$$w_t - p_t = \ln \left( \frac{\overline{W}}{P} \right) + \frac{1}{\gamma} \hat{A}_t - \frac{1-\gamma}{\gamma} \hat{Y}_t. \tag{33}$$

[2]Note that the log price chosen by firm $j$ can be written as $q_{j,t} = q_{j,t}^{*} + d_{j,t}$, where $d_{j,t}$ denotes the distance between the optimal log price and the closest log price point. We observe that $d_{j,t} \sim U\left[-\frac{\Delta_j}{2}, \frac{\Delta_j}{2}\right]$ given that $u_j \sim U[0, \Delta_j[$.

The aggregate dynamics of the economy are therefore identical to the ones of a flex-price economy.

It remains to derive the Phillips curve for the PPSI model. Recall that firms update their information if and only if they are affected by an idiosyncratic shock, which happens with probability $1-\alpha$ in each period.

Let $q_t$ be the average log price of firms that are hit by an idiosyncratic shock in period $t$. Because the individual differences of $q_{j,t}$ from $q^*_{j,t}$ wash out in the aggregation and because the average value of $x_{j,t}$ is always zero, this log price can be written as

$$q_t = \int_0^1 q_{j,t} dj = \int_0^1 q^*_{j,t} dj = \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \widehat{ulc}_t + p_t, \tag{34}$$

where we have utilized (30).

Moreover, we note that firms choose log prices $\mathbb{E}_{t-i}[q_t]$ on average if they were hit by a shock $i$ periods ago. Hence the log price level can be written as

$$\begin{aligned} p_t &= (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \mathbb{E}_{t-i} [q_t] \\ &= (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \mathbb{E}_{t-i} \left[ \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \widehat{ulc}_t + p_t \right], \end{aligned} \tag{35}$$

where we have used (34) to replace $q_t$. Equation (35) is equivalent to the expression obtained in Mankiw and Reis (2002, p. 1300). Therefore it can be used to formulate a sticky-information Phillips curve analogous to the one obtained by them:

$$\begin{aligned} \hat{\pi}_t =& \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \frac{1-\alpha}{\alpha} \widehat{ulc}_t \\ &+ (1-\alpha) \sum_{i=0}^{\infty} (\alpha)^i \, \mathbb{E}_{t-1-i} \left[ \hat{\pi}_t + \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left( \widehat{ulc}_t - \widehat{ulc}_{t-1} \right) \right] \end{aligned}$$

$\square$

# B Optimal Price-setting and the Phillips Curve in the SP Model and the PPSP Model

## B.1 Optimal price-setting

In this section, we examine how firms choose their prices in the SP model and the PPSP model. First, we concentrate on the SP model, which entails conditions that are well-known from other papers (see Ascari and Sbordone (2014) for a survey of the literature on the new Keynesian model with positive trend inflation). In a second step, we explain how the results can be generalized to the PPSP model.

### B.1.1 The firms' profit maximization problem

The profit of a goods producer in period $t$ is given by the difference between revenues and total labor costs

$$\Pi_{j,t} = \frac{Q_{j,t}}{P_t} Y_{j,t} - \frac{W_t}{P_t} N_{j,t}. \tag{36}$$

Goods producers can change their prices only with probability $1 - \alpha$ in every period. In addition, recall that the opportunity to change a price always coincides with the arrival of an idiosyncratic shock $X_{j,t}$. In the absence of a PP restriction firm $j$'s optimization problem in period $t$ is

$$\max_{Q_{j,t}} \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \Lambda_{t,t+i} (\alpha)^i \left[ \left( \frac{Q_{j,t}}{P_{t+i}} \right)^{1-\varepsilon} Y_{t+i} - \frac{W_{t+i}}{P_{t+i}} A_{t+i}^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} \left( \frac{Q_{j,t}}{P_{t+i}} \right)^{-\frac{\varepsilon}{\gamma}} Y_{t+i}^{\frac{1}{\gamma}} \right] \right], \tag{37}$$

where $\Lambda_{t,t+i} = \beta^i \frac{U'(C_{t+i})}{U'(C_t)} = \beta^i \frac{C_t}{C_{t+i}}$ denotes the stochastic discount factor between periods $t$ and $t+i$ and where we have used that the demand for good $j$ is given by $Y_{j,t} = \left( \frac{Q_{j,t}}{P_t} \right)^{-\varepsilon} Y_t$ and that firm $j$'s production function is $Y_{j,t} = A_t X_{j,t} N_{j,t}^{\gamma}$. In addition, we have taken into account that $X_{j,t}$ remains unchanged for the duration of a price spell.

Computing the first-order condition with respect to the individual price $Q_{j,t}$ and simplifying results in the following equation for the optimal price of firm $j$ in period $t$, $Q_{j,t}^*$,

$$\left( Q_{j,t}^* \right)^{\frac{\gamma+\varepsilon(1-\gamma)}{\gamma}} = \frac{\varepsilon}{(\varepsilon-1)\gamma} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\alpha\beta)^i A_{t+i}^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} \frac{W_{t+i}}{P_{t+i}} Y_{t+i}^{\frac{1}{\gamma}} C_{t+i}^{-1} P_{t+i}^{\frac{\varepsilon}{\gamma}}}{\mathbb{E}_t \sum_{i=0}^{\infty} (\alpha\beta)^i Y_{t+i} C_{t+i}^{-1} P_{t+i}^{\varepsilon-1}}. \tag{38}$$

It will be convenient to introduce $Q_t^*$ as the optimal price of a firm whose idiosyncratic productivity is $X_{j,t} = 1$ and that does not have to obey a PP restriction. With this definition, $Q_{j,t}^*$ can be written as

$$Q_{j,t}^* = X_{j,t}^{-\frac{1}{\gamma+\varepsilon(1-\gamma)}} Q_t^*. \tag{39}$$

We can now express the real price of a firm that has idiosyncratic productivity $X_{j,t} = 1$ and that is not subject to a PP restriction, $\frac{Q_t^*}{P_t}$, in the following way

$$\left(\frac{Q_t^*}{P_t}\right)^{\frac{\gamma+\varepsilon(1-\gamma)}{\gamma}} = \frac{\varepsilon}{(\varepsilon-1)\gamma}\frac{\psi_t}{\phi_t}. \tag{40}$$

Variables $\psi_t$ and $\phi_t$ are given by

$$\psi_t := \mathbb{E}_t \sum_{i=0}^{\infty} (\alpha\beta)^i \frac{W_{t+i}}{P_{t+i}} A_{t+i}^{-\frac{1}{\gamma}} Y_{t+i}^{\frac{1}{\gamma}} C_{t+i}^{-1} \pi_{t,t+i}^{\frac{\varepsilon}{\gamma}}, \tag{41}$$

$$\phi_t := \mathbb{E}_t \sum_{i=0}^{\infty} (\alpha\beta)^i Y_{t+i} C_{t+i}^{-1} \pi_{t,t+i}^{\varepsilon-1}, \tag{42}$$

where $\pi_{t,t+i} := P_{t+i}/P_t$. With the help of $\pi_t = P_t/P_{t-1}$, $\psi_t$ and $\phi_t$ can be written recursively:

$$\psi_t = \frac{W_t}{P_t}\frac{Y_t^{\frac{1}{\gamma}}}{A_t^{\frac{1}{\gamma}} C_t} + \alpha\beta\mathbb{E}_t\left(\pi_{t+1}^{\frac{\varepsilon}{\gamma}}\psi_{t+1}\right), \tag{43}$$

$$\phi_t = \frac{Y_t}{C_t} + \alpha\beta\mathbb{E}_t\left(\pi_{t+1}^{\varepsilon-1}\phi_{t+1}\right) \tag{44}$$

Using the definition of unit labor costs, $ulc_t := \frac{W_t N_t}{P_t Y_t}$, we can rewrite (43) as

$$\psi_t = \frac{ulc_t}{s_t}\frac{Y_t}{C_t} + \alpha\beta\mathbb{E}_t\left(\pi_{t+1}^{\frac{\varepsilon}{\gamma}}\psi_{t+1}\right), \tag{45}$$

where we have introduced $s_t := \frac{A_t^{\frac{1}{\gamma}} N_t}{Y_t^{\frac{1}{\gamma}}}$. Variable $s_t$ is viewed as a measure of price dispersion in the literature. In Section B.2, we will detail how it can be computed. To sum up, whenever a firm can adjust its price in the SP model, it selects $Q_{j,t}^*$, where $Q_{j,t}^*$ satisfies (40), (44) and (45).

It is now straightforward to determine the optimal price-setting in the PPSP model. Using the same argument that we applied in our analysis of the PP model, we can conclude that firms that are subject to a PP restriction choose the log price point that is closest to $\log(Q^*_{j,t})$. Thus, subject to a PP restriction, firm $j$'s optimal price can be stated as $Q^*_{j,t}e^{\mu_{j,t}}$, where $\mu_{j,t}$ stands for the deviation of the log price from $\log(Q^*_{j,t})$ that occurs whenever $\log(Q^*_{j,t})$ is not a price point. If the firm's log price points have a distance of $\Delta_j$, then $\mu_{j,t}$ is a number from the interval $[-\Delta_j/2, \Delta_j/2[$.

### B.1.2 Log-linearization

In line with the notation introduced in the main paper, lowercase letters represent the log levels of variables and variables with a "hat" stand for relative deviations from their steady state values.

Log-linearizing (40) yields

$$q^*_t - p_t = \frac{\gamma}{\gamma + \varepsilon(1-\gamma)} \left( \hat{\psi}_t - \hat{\phi}_t \right), \tag{46}$$

where $q^*_t := \log(Q^*_t)$ and $p_t := \log(P_t)$.

Log-linearizing (45) entails

$$\hat{\psi}_t = \left(1 - \alpha\beta\bar{\pi}^{\frac{\varepsilon}{\gamma}}\right) \left[\widehat{ulc}_t - \hat{s}_t + \hat{Y}_t - \hat{C}_t\right] + \alpha\beta\bar{\pi}^{\frac{\varepsilon}{\gamma}} \left[\mathbb{E}_t\hat{\psi}_{t+1} + \frac{\varepsilon}{\gamma}\mathbb{E}_t\hat{\pi}_{t+1}\right]. \tag{47}$$

Equation (44) can be approximated as

$$\hat{\phi}_t = (1 - \alpha\beta\bar{\pi}^{\varepsilon-1})(\hat{Y}_t - \hat{C}_t) + \alpha\beta\bar{\pi}^{\varepsilon-1} \left[\mathbb{E}_t\hat{\phi}_{t+1} + (\varepsilon - 1)\mathbb{E}_t\hat{\pi}_{t+1}\right]. \tag{48}$$

Hence in the SP model the log price chosen by firm $j$ is given by $-\frac{1}{\gamma+\varepsilon(1-\gamma)}x_{j,t} + \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\left(\hat{\psi}_t - \hat{\phi}_t\right) + p_t$, where $\hat{\psi}_t$ and $\hat{\phi}_t$ can be obtained from (47) and (48). For the PPSP model, the respective log price is given by $-\frac{1}{\gamma+\varepsilon(1-\gamma)}x_{j,t} + \frac{\gamma}{\gamma+\varepsilon(1-\gamma)}\left(\hat{\psi}_t - \hat{\phi}_t\right) + p_t + \mu_{j,t}$.

## B.2 Price dispersion

### B.2.1 A recursive expression for price dispersion

It is common to interpret the variable $s_t$ introduced in the previous subsection as a measure of price dispersion. As is well-known, the log deviation of $s_t$ from its state-state level is zero for a log-linear approximation around a zero-inflation steady state. As a consequence, this term is often ignored in new Keynesian analyses.

As we allow for positive trend inflation, we need to examine $s_t$ more closely. In this subsection, we focus on price dispersion in the PPSP model. By setting, $\mu_{j,t}$ to zero for all firms, it is straightforward to obtain expressions for $s_t$ in the SP model. It will be useful to introduce $\mathcal{Q}_{j,t} := X_{j,t}^{\frac{1}{\gamma+\varepsilon(1-\gamma)}} Q_{j,t} e^{-\mu_{j,t}}$, i.e. $\mathcal{Q}_{j,t}$ is the hypothetical price of firm $j$ if its idiosyncratic productivity was $X_{j,t} = 1$ and if $\mu_{j,t} = 0$, which would imply that the optimal price in the absence of the PP restriction exactly corresponds to a price point.

Aggregate labor can be written in the following way:

$$\begin{aligned} N_t &= \int_0^1 N_{j,t} dj = \int_0^1 \left( \frac{Y_{j,t}}{A_t X_{j,t}} \right)^{\frac{1}{\gamma}} dj = Y_t^{\frac{1}{\gamma}} \int_0^1 A_t^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} \left( \frac{Q_{j,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} dj \\ &= Y_t^{\frac{1}{\gamma}} A_t^{-\frac{1}{\gamma}} \int_0^1 X_{j,t}^{-\frac{1}{\gamma}} \left( \frac{X_{j,t}^{-\frac{1}{\gamma+\varepsilon(1-\gamma)}} \mathcal{Q}_{j,t} e^{\mu_{j,t}}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} dj \\ &= Y_t^{\frac{1}{\gamma}} A_t^{-\frac{1}{\gamma}} \underbrace{\widetilde{\omega} \int_0^1 \left( \frac{\mathcal{Q}_{j,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} dj}_{s_t}, \end{aligned} \tag{49}$$

where $\widetilde{\omega} := \int_0^1 X_{j,t}^{\frac{\varepsilon-1}{\gamma+\varepsilon(1-\gamma)}} dj \cdot \int_0^1 e^{-\frac{\varepsilon}{\gamma}\mu_{j,t}} dj$, which is constant over time, as the distributions of idiosyncratic productivities and of the distances of optimal prices from price points are stationary.

Given our assumption of Calvo price stickiness and the observation that $\mathcal{Q}_{j,t} = Q_t^*$ holds for firms that adjust their prices in period $t$, $s_t$ satisfies

$$\begin{aligned} s_t =& \widetilde{\omega} \int_0^1 \left( \frac{\mathcal{Q}_{j,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} dj \\ =& (1-\alpha) \left( \frac{Q_t^*}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} \widetilde{\omega} + \alpha \pi_t^{\frac{\varepsilon}{\gamma}} s_{t-1}. \end{aligned} \tag{50}$$

As a next step, we need to examine how the price level evolves over time. Under Calvo pricing, the price level

$$P_t = \left[\int_0^1 (Q_{j,t})^{1-\varepsilon}\, dj\right]^{\frac{1}{1-\varepsilon}} \tag{51}$$

can be written as

$$P_t = \left[\alpha P_{t-1}^{1-\varepsilon} + (1-\alpha)\omega\, (Q_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{52}$$

where we have used that $Q_{j,t}^* = X_{j,t}^{-\frac{1}{\gamma+\varepsilon(1-\gamma)}} Q_t^*$ and introduced $\omega := \int_0^1 X_{j,t}^{\frac{\varepsilon-1}{\gamma+\varepsilon(1-\gamma)}} dj \cdot \int_0^1 e^{(1-\varepsilon)\mu_{j,t}}\, dj$, which is constant over time, as the distributions of idiosyncratic productivities and of the distances of optimal log prices from the closest log price points are stationary.

Dividing (52) by $P_t$ yields

$$1 = \left[\left(\alpha P_{t-1}^{1-\varepsilon} + (1-\alpha)\omega\, (Q_t^*)^{1-\varepsilon}\right) P_t^{-(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}, \tag{53}$$

$$1 = \alpha \pi_t^{\varepsilon-1} + (1-\alpha)\omega \left(\frac{Q_t^*}{P_t}\right)^{1-\varepsilon}, \tag{54}$$

$$\frac{Q_t^*}{P_t} = \left[\frac{1-\alpha\pi_t^{\varepsilon-1}}{(1-\alpha)\omega}\right]^{\frac{1}{1-\varepsilon}}. \tag{55}$$

### B.2.2 Log-linearization

Log-linearizing (50) yields

$$\hat{s}_t = -\frac{\varepsilon}{\gamma}\left(1-\alpha\bar{\pi}^{\frac{\varepsilon}{\gamma}}\right)(q_t^* - p_t) + \alpha\bar{\pi}^{\frac{\varepsilon}{\gamma}}\left(\frac{\varepsilon}{\gamma}\hat{\pi}_t + \hat{s}_{t-1}\right). \tag{56}$$

Log-linearizing (55) entails

$$q_t^* - p_t = \frac{\alpha\overline{\pi}^{\varepsilon-1}}{1-\alpha\overline{\pi}^{\varepsilon-1}}\hat{\pi}_t, \tag{57}$$

where $\hat{\pi}_t$ is the log deviation of inflation from its long-run gross level $\overline{\pi}$. Combining (56) and (57) yields the following recursive expression for $\hat{s}_t$, which can be used to determine $\hat{s}_t$ in (47), when we calculate how firms select prices:

$$\hat{s}_t = \frac{\varepsilon}{\gamma}\frac{\overline{\pi}^{\frac{\varepsilon}{\gamma}} - \overline{\pi}^{\varepsilon-1}}{1-\alpha\overline{\pi}^{\varepsilon-1}}\alpha\widehat{\pi}_t + \alpha\overline{\pi}^{\frac{\varepsilon}{\gamma}}\hat{s}_{t-1} \tag{58}$$

We observe that for a zero-inflation steady state, i.e. $\bar{\pi} = 1$, (58) has the solution $\hat{s}_t = 0$. While (58) has been derived for the PPSP model, it holds for the SP model as well. This can be verified easily by assuming that $\mu_{j,t} = 0$ for all firms.

### B.3 New Keynesian Phillips curve

#### B.3.1 Price level, aggregation and price dispersion

It is straightforward to combine (55) with (40), (44), and (45) to obtain a variant of the non-linear new Keynesian Phillips curve for the PPSP model.

#### B.3.2 Log-linearization

In the paper, we consider only a log-linearized version of this Phillips curve. Together with (46), (57) yields the condition

$$\hat{\pi}_t = \frac{1-\alpha\overline{\pi}^{\varepsilon-1}}{\alpha\overline{\pi}^{\varepsilon-1}} \frac{\gamma}{\gamma+\varepsilon(1-\gamma)} \left(\hat{\psi}_t - \hat{\phi}_t\right), \tag{59}$$

where $\hat{\psi}_t$ and $\hat{\phi}_t$ are given by (47) and (48). An identical Phillips curve holds for the SP model. It may be noteworthy that the parameter $\alpha$ is calibrated to different values in the two models.

## C Relationship to Menu-Cost Models

In the paper, we use a model with Calvo pricing as the main benchmark for our comparisons but one might also ask how the PPSI model would fare against a variant with menu costs. While a rigorous analysis of such a model variant is beyond the scope of this paper, we offer a few thoughts about the relationship between our PP restriction and menu costs.

In some respects, menu costs and a PP restriction lead to similar predictions. For example, in the absence of idiosyncratic and aggregate shocks, both modeling approaches entail that all prices move upwards in a step-wise manner under positive trend inflation. Due to the selection effect, which involves that only the prices farthest away from their optimal values adjust, basic menu-cost models predict that changes in money growth rates have no effect on real variables (see Caplin and Spulber (1987)). This is closely related to the observation that our model with a PP restriction but without information frictions would imply that monetary policy has purely nominal effects. Moreover, some empirical findings like the larger sizes of price decreases compared to increases could be explained by menu cost models as well (see NS).

However, menu costs and our PP restriction do not always lead to identical predictions. Menu costs involve that relatively small price changes within a certain interval do not occur but that all price changes from a continuum outside this interval may occur.[3] By contrast, a PP restriction requires that all price changes come in discrete steps. It is because of this difference that Knotek (2016) finds that price points are more relevant for understanding price dynamics than menu costs.

Finally, it is well-known that jumps in the price level would induce many firms to adjust their prices simultaneously in a menu-cost model (see Caplin and Spulber (1987, p. 720)). At least temporarily, these coordinated price changes would substantially reduce any price dispersion that is not driven by differences in productivities or similar fundamental factors.[4] This arguably implausible effect does not occur under a PP restriction.

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[3]For stochastic menu costs, the length of this interval may not be constant.

[4]In a similar vein, menu cost models would predict that the introduction of a new currency like the Euro in many European countries would lead to a temporary reduction in price dispersion.

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