Internet Appendix to:

Cuesdeanu, Horatio, and Jens Jackwerth, 2018, The Pricing Kernel Puzzle: Survey and Outlook, *Annals of Finance*, forthcoming.

Internet Appendix.KMM

We derive the pricing kernel for our ambiguity aversion model as follows. Introducing the Lagrange multiplier λ, we write the N times M first order conditions

$$p_j\ \phi'\left(\sum_{k=1}^{N} p_{kj}U(C_{kj})\right)p_{ij}U'(C_{ij}) - \pi_{ij}\lambda = 0 \text{ for i} = 1, \dots, \text{N and for j} = 1, \dots, \text{M} \quad \text{(KMM.1)}$$

and solve for λ by summing equations (KMM.1) over j and i. We make the simplifying assumption that there exists a risk-free asset with $R_f(= 1\ /\ \sum_{j=1}^{M}\sum_{i=1}^{N}\pi_{ij})$ being one plus the risk-free rate.

$$R_f \sum_{j=1}^{M}\sum_{i=1}^{N} p_j\ \phi'\left(\sum_{k=1}^{N} p_{kj}U(C_{kj})\right)p_{ij}U'(C_{ij}) = \lambda \quad \text{(KMM.2)}$$

Substituting λ back into equation (KMM.1) and summing over j, we obtain after rearranging

$$\pi_i = \sum_{j=1}^{M}\pi_{ij} = \frac{\sum_{j=1}^{M} p_j\ \phi'\left(\sum_{k=1}^{N} p_{kj}U(C_{kj})\right)p_{ij}U'(C_{ij})}{R_f\sum_{j=1}^{M}\sum_{s=1}^{N} p_j\ \phi'\left(\sum_{s=1}^{N} p_{kj}U(C_{kj})\right)p_{sj}U'(C_{sj})} \text{for i} = 1, \dots, \text{N} \quad \text{(KMM.3)}$$

We can now solve for the pricing kernel m_i as the ratio of π_i and expected physical probabilities in state i, where we take the expectations across ambiguity settings j:

$$m_i = \frac{\pi_i}{\sum_{j=1}^{M} p_j p_{ij}} = \frac{1}{\sum_{j=1}^{M} p_j p_{ij}}\ \frac{\sum_{j=1}^{M} p_j\ \phi'\left(\sum_{k=1}^{N} p_{kj}U(C_{kj})\right)p_{ij}U'(C_{ij})}{R_f\sum_{j=1}^{M}\sum_{s=1}^{N} p_j\ \phi'\left(\sum_{k=1}^{N} p_{kj}U(C_{kj})\right)p_{sj}U'(C_{sj})} \text{ for i} = 1, \dots, \text{N} \quad \text{(KMM.4)}$$

In equilibrium, consumption needs to equal the return on the market in each state i, thus $\boldsymbol{C_{ij}} = \boldsymbol{w_0 R_i}$. We thus express the pricing kernel as:

$$m_i = \frac{1}{\sum_{j=1}^{M} p_j p_{ij}} \frac{\sum_{j=1}^{M} p_j \, \phi'\left(\sum_{k=1}^{N} p_{kj} U(w_0 R_k)\right) p_{ij} U'(w_0 R_i)}{R_f \sum_{j=1}^{M} \sum_{s=1}^{N} p_j \, \phi'\left(\sum_{k=1}^{N} p_{kj} U(w_0 R_k)\right) p_{sj} U'(w_0 R_s)} \text{ for i} = 1, \ldots, \text{N} \quad \text{(KMM. 5)}$$

Next, we implement two versions of our ambiguity aversion model. We note that we could have chosen an alternative utility function U(x), namely, $U(x) = \frac{x^{1-\gamma} - 1}{1-\gamma}$ with $\gamma \in (0,1)$ but the formulation in the main paper allows for a greater range of risk aversion coefficients.

In the model with ambiguity over volatility, we model the 30-day return being lognormally distributed with an annualized mean of 0.10. There are 300 log return levels from -0.99 to +2.00 in steps of 0.01. The investors are ambiguous with respect to annualized volatility, which we assume to be lognormally distributed with mean log 0.19 and standard deviation 0.10. There are 81 ambiguity settings ranging from -4 to +4 standard deviations in steps of 0.1 standard deviations. It is interesting to note that, contrary to common models using power utilities, here it does matter how we specify the utility function U; by not subtracting 1 in the numerator, we do not obtain the pricing kernel puzzle of Figure 7 with these parameters.

In the model with ambiguity over jumps, we introduce large negative jumps (-0.20 annualized mean and 0.30 standard deviation) where the investor exhibits ambiguity aversion across the probability of such jumps occurring. Here, we assume a uniform distribution from 0 to 0.5 in 81 equally spaced steps of 0.0063. The return distribution without crashes is modeled being lognormally distributed with an annualized mean return of 0.12 and a volatility of 0.19. There are again 300 log return levels from -0.99 to +2.00 in steps of 0.01. Finally, the conditional probabilities p_{ij} are obtained by mixing the return distribution without the crashes with the jump distribution. The probabilities for the occurrence of a jump then determine the appropriate weights for the two distributions such that the p_{ij} add up to 1 for a fixed j.